



Scalable full quantum dynamics of dissipative Bose-Hubbard systems and multi-time correlations



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PRX Quantum 2, 010319 (2021)
Quantum 5, 455 (2021)

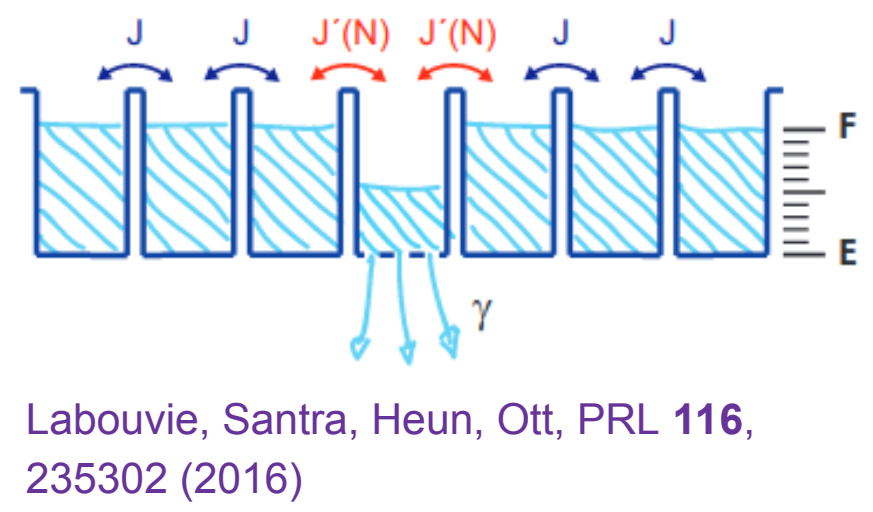
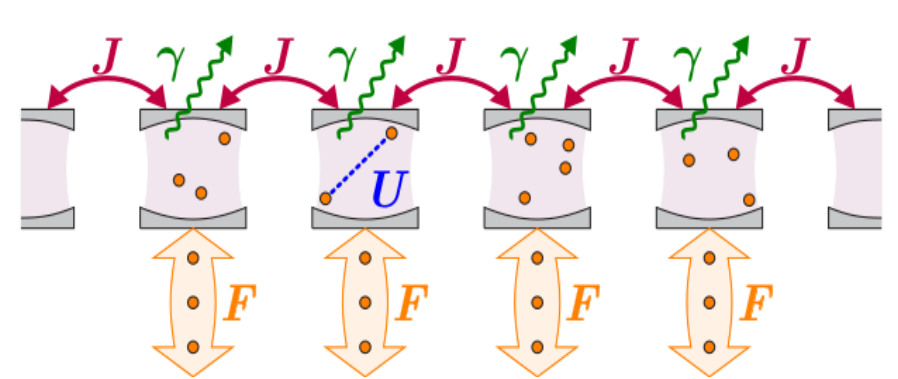
Conclusion: full quantum dynamics of up to millions of sites can be done in the right parameter ranges

Dissipative Bose-Hubbard model

$$\hat{H} = \sum_j \hat{H}_j - \sum_{\text{connections } i,j} [J_{ij} \hat{a}_j^\dagger \hat{a}_i + J_{ij}^* \hat{a}_i^\dagger \hat{a}_j]$$

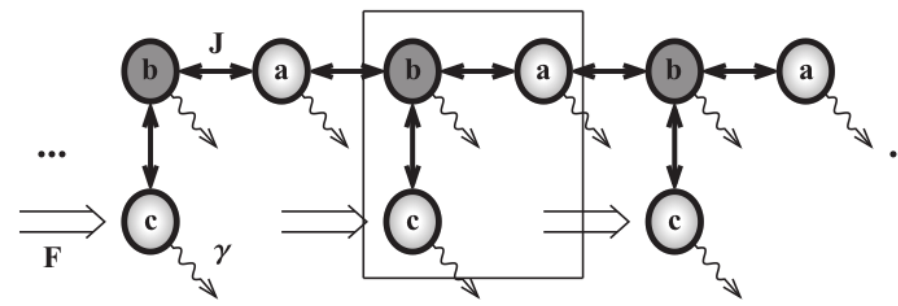
$$\hat{H}_j = -\Delta_j \hat{a}_j^\dagger \hat{a}_j + \frac{U_j}{2} \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j + F_j \hat{a}_j^\dagger + F_j^* \hat{a}_j$$

$$\frac{\partial \hat{\rho}}{\partial t} = -i [\hat{H}, \hat{\rho}] + \sum_j \frac{\gamma_j}{2} [2\hat{a}_j \hat{\rho} \hat{a}_j^\dagger - \hat{a}_j^\dagger \hat{a}_j \hat{\rho} - \hat{\rho} \hat{a}_j^\dagger \hat{a}_j]$$



Vincentini, Minganti, Rota, Orso, Ciuti, PRA 97, 013853 (2018)

Labouvie, Santra, Heun, Ott, PRL 116, 235302 (2016)



Casteels, Rota, Storme, Ciuti, PRA 93, 043833 (2016)

Baboux, Ge, Jacquini, Biondi, Galopin, Lemaître, Le Gratiet, Sagnes, Schmidt, Tureci, Amo, Bloch, PRL 116, 066402 (2016)

Positive-P representation

M subsystems (modes, sites, volumes) labeled by j

Coherent state basis, complex, local α_j $|\alpha_j\rangle_j = e^{-|\alpha_j|^2/2} e^{\alpha_j \hat{a}_j^\dagger} |\text{vac}\rangle$

Local operator kernel $\hat{\Lambda}(\lambda) = \bigotimes_j \frac{|\alpha_j\rangle_j \langle \beta_j^*|_j}{\langle \beta_j^*|_j \langle \alpha_j|_j}$

full system configuration $\lambda = \{\alpha_1, \dots, \alpha_M, \beta_1, \dots, \beta_M\}$

Full density matrix $\hat{\rho} = \int d^M \lambda P_+(\lambda) \hat{\Lambda}(\lambda)$

Correlations between subsystems are all in the distribution of configurations

$P_+(\lambda)$ The distribution is positive, real \rightarrow let's SAMPLE IT!

Quantum dynamics (1-mode example):

Density matrix $\hat{\rho} \leftrightarrow$ distribution P_+ for the fields \leftrightarrow random samples of the fields α, β

Master equation: $\hat{H} = \frac{U}{2} \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} - \Delta \hat{a}^\dagger \hat{a}$ $\hbar = 1$

$\frac{\partial \hat{\rho}}{\partial t} = -i [\hat{H}, \hat{\rho}] + \frac{\gamma}{2} (2\hat{a} \hat{\rho} \hat{a}^\dagger - \hat{a}^\dagger \hat{a} \hat{\rho} - \hat{\rho} \hat{a}^\dagger \hat{a})$ dissipation γ

Fokker Planck equation $\frac{\partial P_+}{\partial t} = \left\{ -\frac{\partial}{\partial \alpha} (-iU\alpha\beta + i\Delta - \frac{\gamma}{2}) \alpha - \frac{\partial}{\partial \beta} (iU\alpha\beta - i\Delta - \frac{\gamma}{2}) \beta + \frac{\partial^2}{\partial \alpha^2} \left(\frac{-iU}{2} \alpha^2 + \frac{\partial^2}{\partial \beta^2} \left(\frac{iU}{2} \beta^2 \right) \right) \right\} P_+$

Stochastic (Langevin) equations: $\frac{d\alpha}{dt} = (-iU\alpha\beta + i\Delta - \frac{\gamma}{2}) \alpha + \sqrt{-iU} \alpha \xi(t)$ $\frac{d\beta}{dt} = (+iU\alpha\beta - i\Delta - \frac{\gamma}{2}) \beta + \sqrt{+iU} \beta \xi(t)$

Dissipative Bose-Hubbard dynamics:

$\frac{\partial \alpha_j}{\partial t} = i\Delta_j \alpha_j - iU_j \alpha_j^2 \alpha_j^* - iF_j - \frac{\gamma_j}{2} \alpha_j + \sqrt{-iU_j} \alpha_j \xi_j(t) + \sum_k iJ_{kj} \alpha_k$

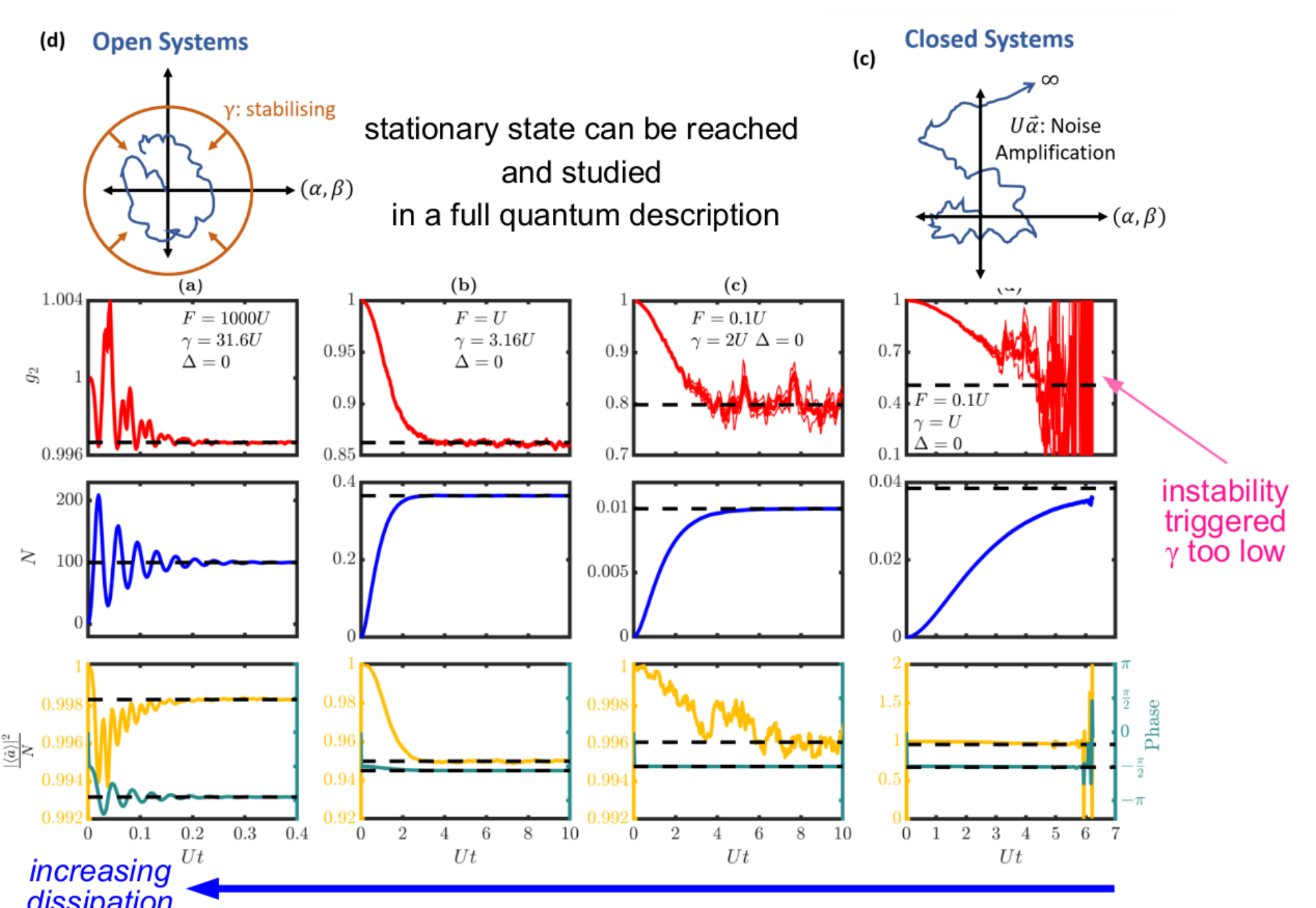
$\frac{\partial \tilde{\alpha}_j}{\partial t} = i\Delta_j \tilde{\alpha}_j - iU_j \tilde{\alpha}_j^2 \tilde{\alpha}_j^* - iF_j - \frac{\gamma_j}{2} \tilde{\alpha}_j + \sqrt{-iU_j} \tilde{\alpha}_j \tilde{\xi}_j(t) + \sum_k iJ_{kj} \tilde{\alpha}_k$

White Gaussian noise deals with interparticle collisions

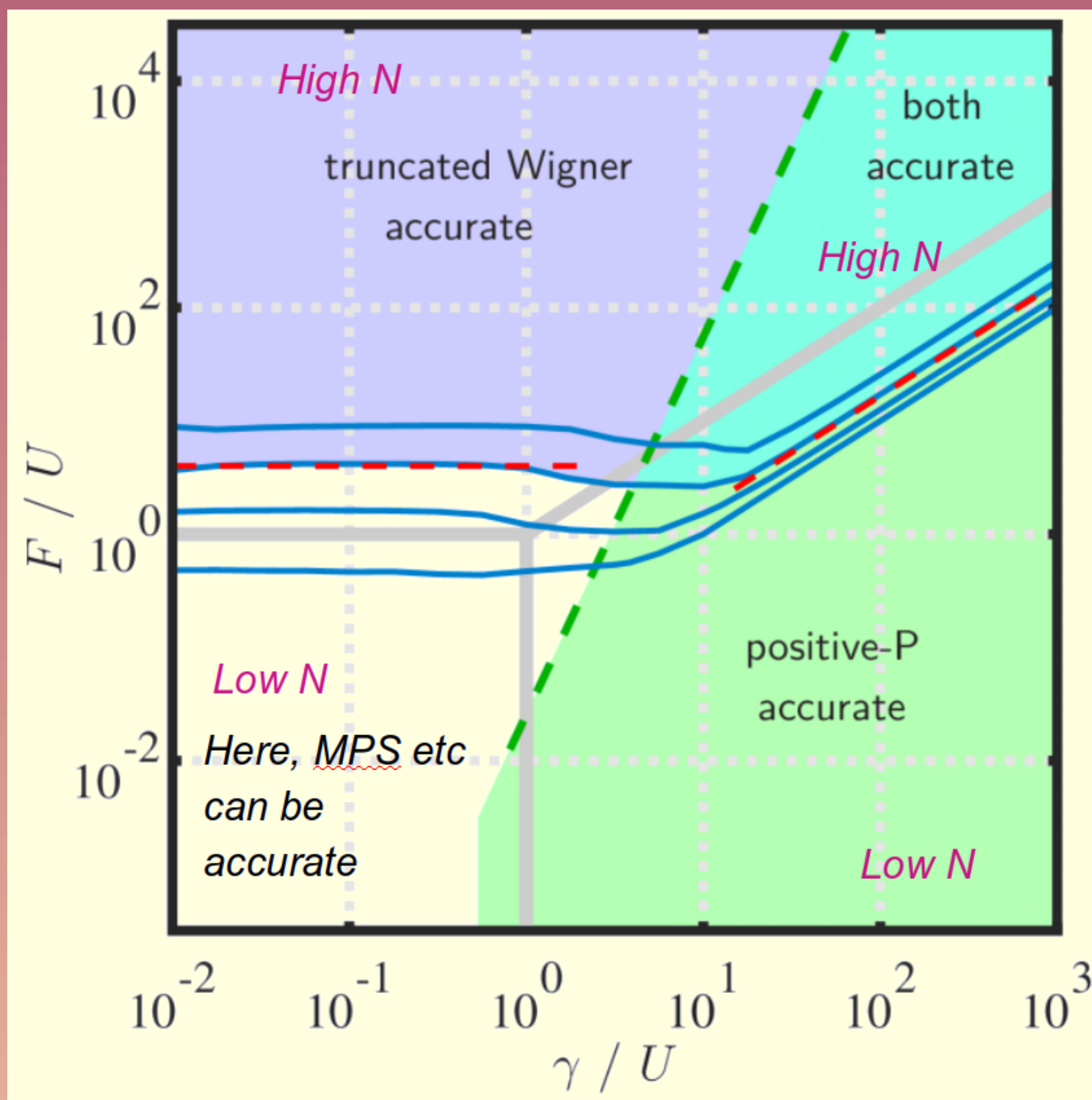
$\langle \xi_j(t) \xi_k(t') \rangle_s = \delta(t-t') \delta_{jk}$, $\langle \tilde{\xi}_j(t) \tilde{\xi}_k(t') \rangle_s = \delta(t-t') \delta_{jk}$

The rest of the equations is basically mean field

Stabilisation by dissipation - 1 mode



REGIONS OF APPLICABILITY



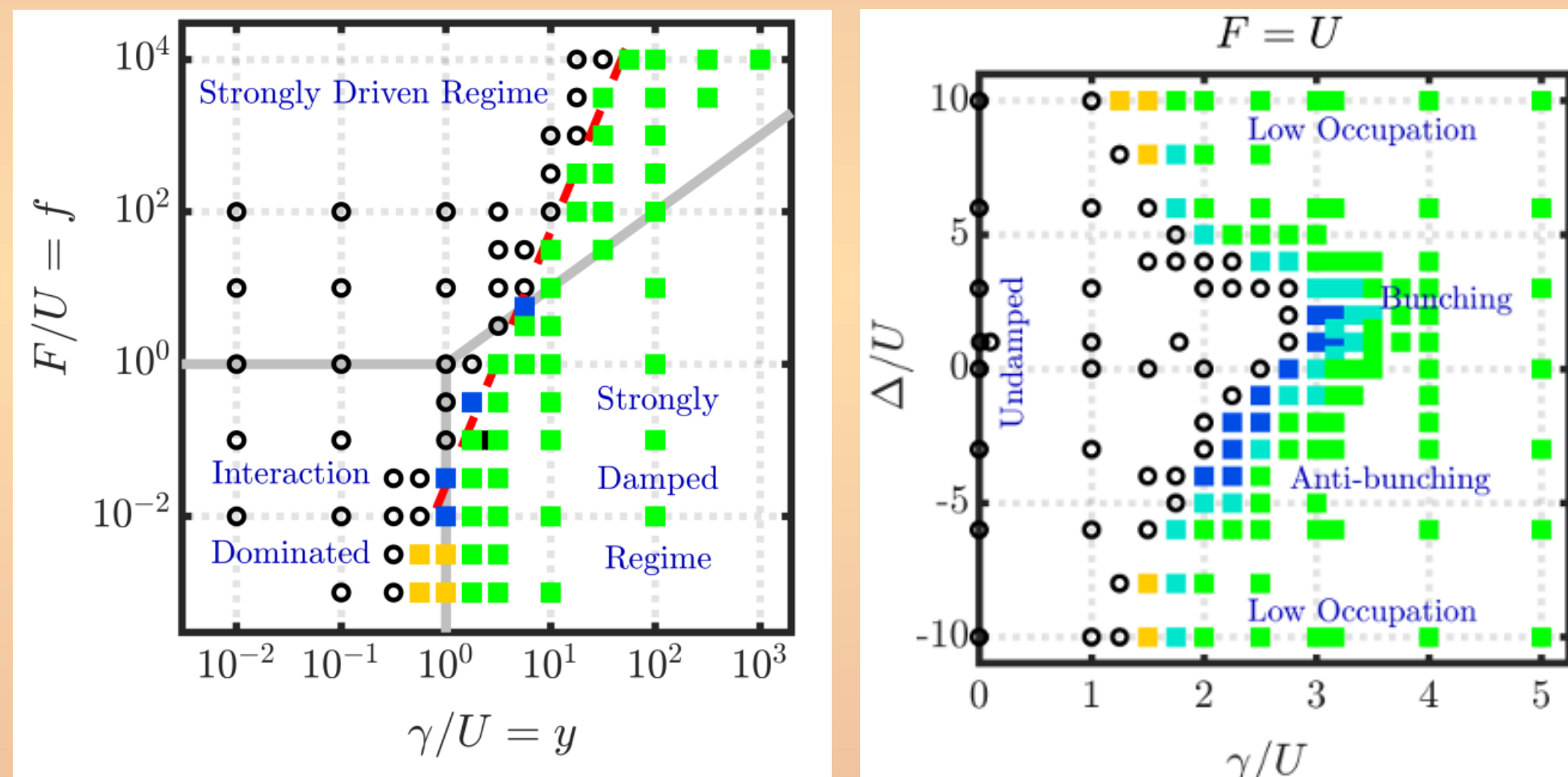
Positive-P stability:

determined by single site parameters

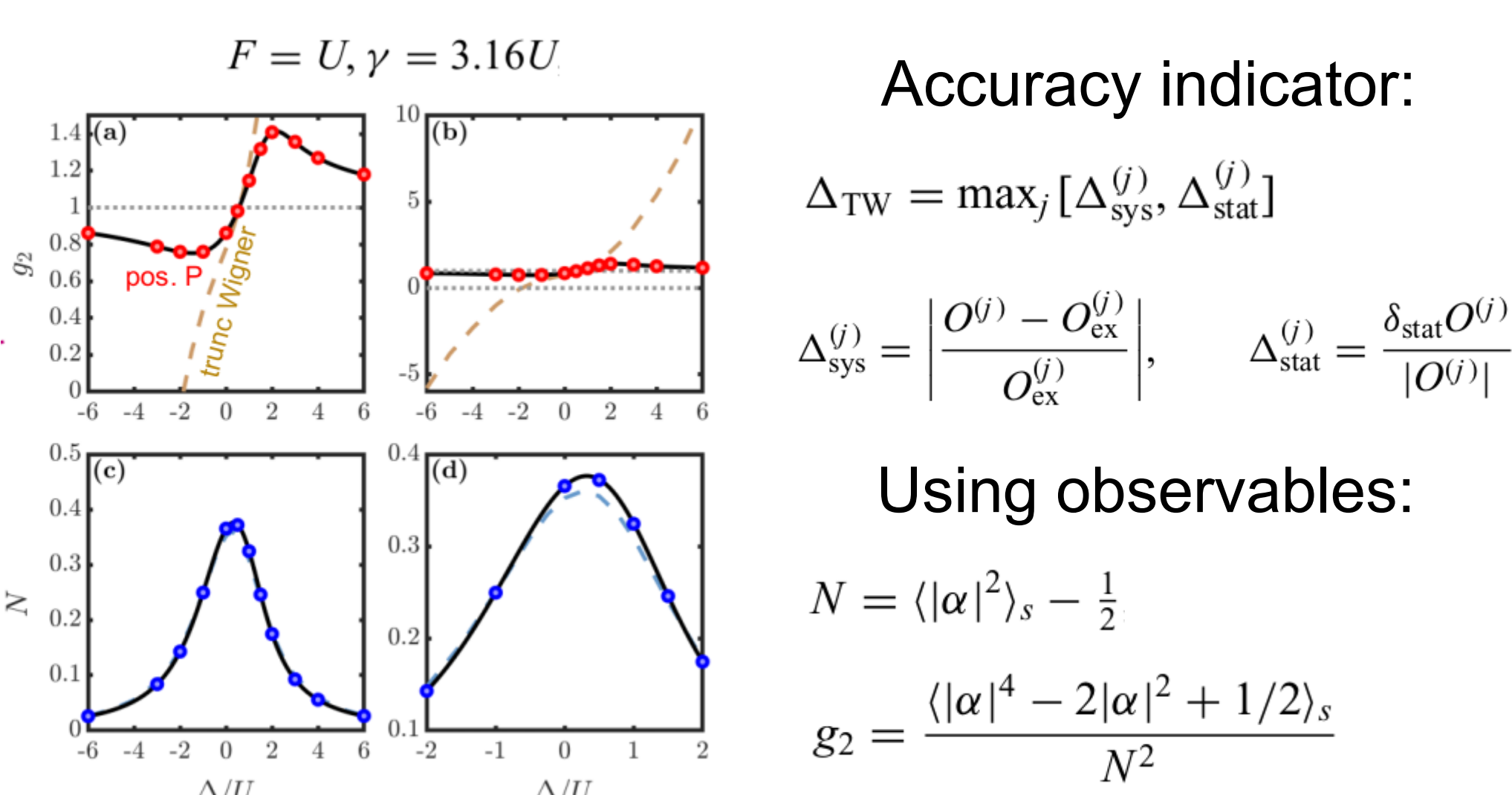
$\gamma \gtrsim 3U \left(\frac{F}{U} \right)^{0.30}$ resultant occupation dependence of stability region

$N \sim \left(\frac{F}{U} \right)^{2/3}$ $\gamma \gtrsim 3U \sqrt{N}$

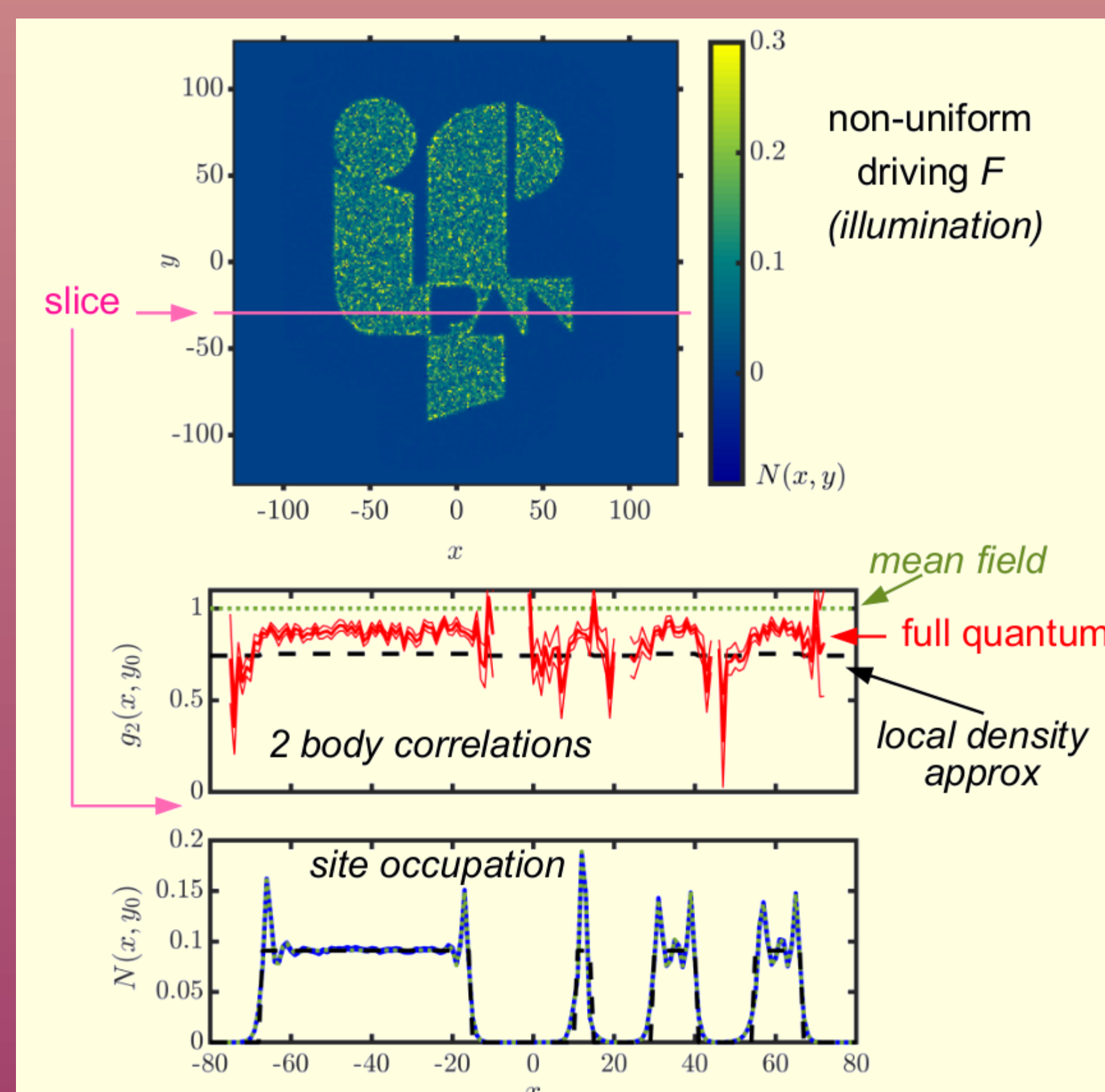
Single mode testing:



truncated Wigner (in)accuracy



Large nonuniform system 256 x 256 sites



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Multi-time correlations

Expressed in terms of Heisenberg operators

$\hat{A}(t) = e^{i(t-t_0)\hat{H}/\hbar} \hat{A}(t_0) e^{-i(t-t_0)\hat{H}/\hbar}$

Time ordered correlation functions. Correspond to all sequences of measurements

$\langle \hat{A}_1(t_1) \hat{A}_2(t_2) \dots \hat{A}_N(t_N) \hat{B}_1(s_1) \hat{B}_2(s_2) \dots \hat{B}_M(s_M) \rangle$ $t_1 \leq t_2 \leq \dots \leq t_N$
 $s_1 \geq s_2 \geq \dots \geq s_M$

Heisenberg equations of motion:

$\frac{d\hat{a}(t)}{dt} = (-iU\hat{a}^\dagger(t)\hat{a}(t) + i\Delta - \frac{\gamma}{2}) \hat{a}(t)$
 $\frac{d\hat{a}^\dagger(t)}{dt} = \hat{a}^\dagger(t) (+iU\hat{a}^\dagger(t)\hat{a}(t) - i\Delta - \frac{\gamma}{2})$

positive-P equations of motion

$\frac{d\alpha}{dt} = (-iU\alpha\beta + i\Delta - \frac{\gamma}{2}) \alpha + \sqrt{-iU} \alpha \xi(t)$
 $\frac{d\beta}{dt} = (+iU\alpha\beta - i\Delta - \frac{\gamma}{2}) \beta + \sqrt{+iU} \beta \xi(t)$

Notice similarity

Main result

Normal ordering: positive-P variables

$\langle \hat{a}_{p_1}^\dagger(t_1) \dots \hat{a}_{p_N}^\dagger(t_N) \hat{a}_{q_1}(s_1) \dots \hat{a}_{q_M}(s_M) \rangle = \langle \beta_{p_1}(t_1) \dots \beta_{p_N}(t_N) \alpha_{q_1}(s_1) \dots \alpha_{q_M}(s_M) \rangle_{\text{stoch}}$

Anti-normal ordering: Q distribution variables

$\langle \hat{a}_{p_1}(t_1) \dots \hat{a}_{p_N}(t_N) \hat{a}_{q_1}^\dagger(s_1) \dots \hat{a}_{q_M}^\dagger(s_M) \rangle = \langle \alpha'_{p_1}(t_1) \dots \alpha'_{p_N}(t_N) \beta'_{q_1}(s_1) \dots \beta'_{q_M}(s_M) \rangle_{\text{stoch}}$

conversion P \rightarrow Q

$\alpha'_j = \alpha_j + \zeta_j$; $\beta'_j = \beta_j + \zeta_j^*$

$\langle \zeta_j \rangle_{\text{stoch}} = 0$; $\langle \zeta_j \zeta_k \rangle_{\text{stoch}} = 0$; $\langle \zeta_j^* \zeta_k \rangle_{\text{stoch}} = 1$

Mixed ordering:

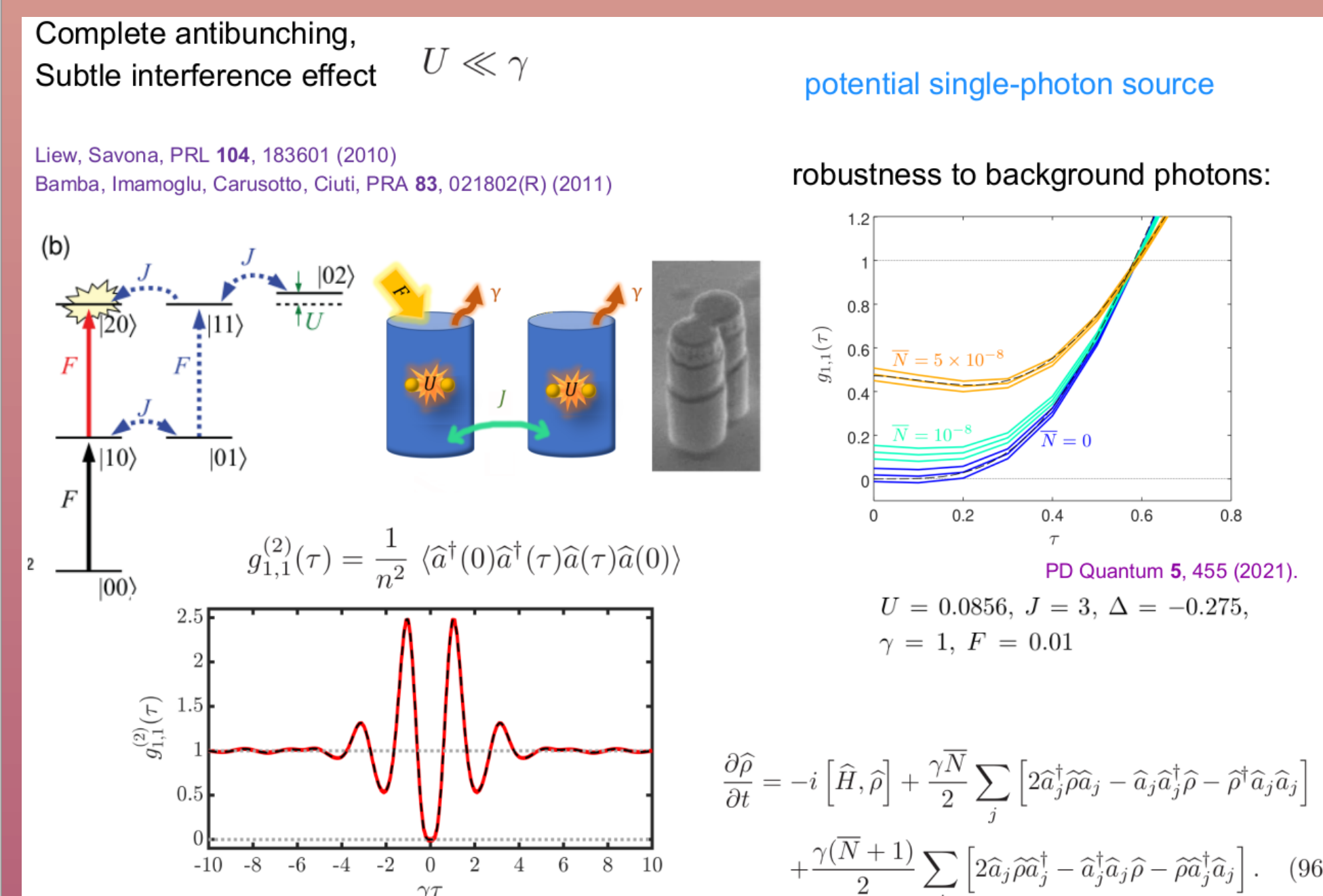
- 1) sample what is possible using positive-P variables
- 2) convert variables to doubled-Q
- 3) sample what is possible using Q variables

Coverage

Order (number of operators)	2nd order	3rd order	4th order
Total permutations	12	56	240
single time correlations	4	8	16
multi-time accessible with P representation	4	14	36
additional accessible with Q representation	4	14	36
additional accessible with mixed order (Sec. 5.4)	-	12	72
Total doable	12	48	160
time ordered not doable	-	-	6
Not time ordered, not doable	-	-	24
Not time ordered, doable	-	8	80

Table 1: A tally of \hat{a}, \hat{a}^\dagger product permutations that can/cannot be evaluated with the various approaches discussed. The general form considered is $\langle \hat{A}(t_1) \hat{B}(t_2) \hat{C}(t_3) \dots \rangle$, where $\hat{A}, \hat{B}, \hat{C}$ can be either of \hat{a} or \hat{a}^\dagger (same mode), and the time arguments can take up to three distinct times $t_1 \leq t_2 \leq t_3$. Permutations with the same time topology (e.g. $\hat{A}(t_1) \hat{B}(t_1) \hat{C}(t_2)$ and $\hat{A}(t_1) \hat{B}(t_2) \hat{C}(t_3)$) are counted only once.

Unconventional photon blockade



Have a dissipative system you want to simulate?

non-uniform? time-dependent??

Contact us ;-)