CONTINOUOS OBSERVATION OF A QUANTUM SYSTEM

Filip Gampel, Mariusz Gajda

Institute of Physics, Polish Academy of Sciences

NATIONAL SCIENCE CENTRE POLAND

1. THE MCWF-METHOD

In quantum mechanics, a large class of dissipative processes can be described by the master equation

$$\dot{\rho_S} = \frac{i}{\hbar}\left[\rho_S, H_S\right] + \mathcal{L}_{relax}$$

Where ρ_S is the density matrix and H_S the Hamiltonian of the system. For Markovian processes, the most general form of the relaxation operator in this equation is

$$\mathcal{L}_{relax} = -\frac{1}{2}\sum_m \left(C_m^\dagger C_m \rho_S + \rho_S C_m^\dagger C_m\right) + \sum_m C_m \rho_S C_m^\dagger$$

The C_m are operators which for instance can describe the coupling of the system to a reservoir. Instead of solving the master equation, we pursue an alternative approach due to Castin, Dalibard and Molmer[1]. In this formalism, one obtains single possible trajectories which are given by randomly ocurring „quantum jumps" and evolution with a non-hermitian Hamiltonian inbetween.

This Hamiltonian is given by

$$H = H_S - \frac{i\hbar}{2}\sum_m C_m^\dagger C_m$$

After a small time step δt the wavefunction is no longer normalized, its norm given by 1- δp, with

$$\delta p = \sum_m \delta p_m \quad \text{and} \quad \delta p_m = \delta t \langle \phi(t)|C_m^\dagger C_m|\phi(t)\rangle$$

These δp_m are the probabilities of the respective quantum jumps. To obtain the wavefunction after a timestep, we first randomly choose whether a jump occurs. If so, the new wavefunction is given by

$$|\phi(t+\delta t)\rangle = \frac{C_m|\phi(t)\rangle}{\| C_m|\phi(t)\rangle \|}$$

If not, we just evolve using our Hamiltonian and normalize:

$$|\phi(t+\delta t)\rangle = \frac{|\phi^{(1)}(t+\delta t)\rangle}{\| |\phi^{(1)}(t+\delta t)\rangle \|} \qquad |\phi^{(1)}(t+\delta t)\rangle = \left(1 - \frac{iH\delta t}{\hbar}\right)|\phi(t)\rangle$$

2. OUR MODEL

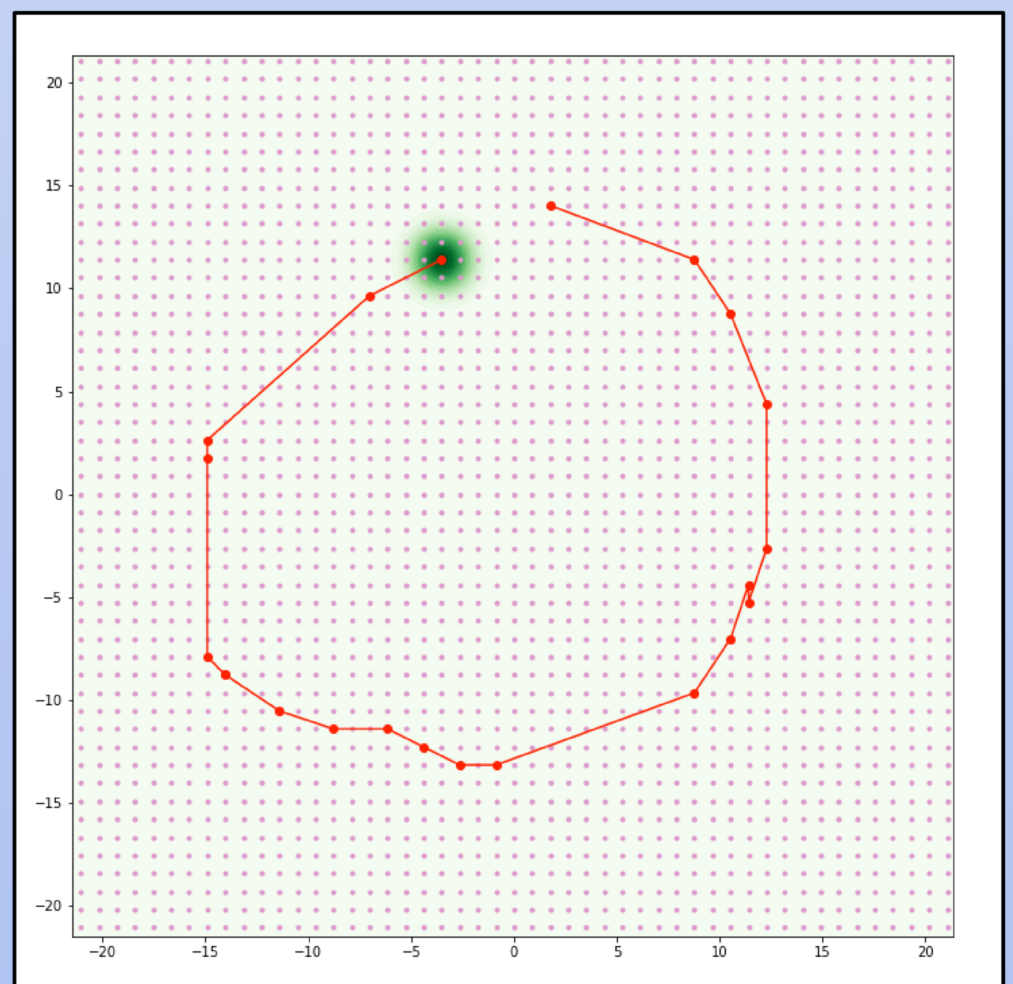
We want to study the behaviour of a system under continuous measurement. In the simplest case, this might be a single particle:

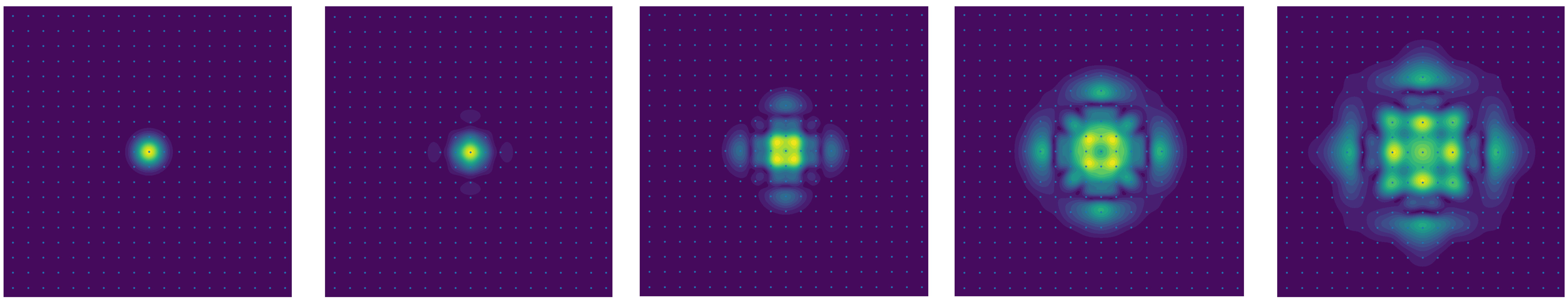
$$H_S = \frac{p^2}{2m} + V$$

The particle is thought to evolve in the 2D plane on a grid of detectors which can measure both position and momentum. In the formalism described above, this means that the C_m are projection operators on localized states with a defined momentum, e.g. Gaussian wavepackets. Each C_m corresponds to a detector, and each grid point accommodates a number of detectors for states with identical position but different momentum.

Right: Trajectory of a particle in a harmonic oscillator with some initial momentum. Red dots signify detectors in which the particle was measured and are connected for improved readability.

Bottom: Evolution of the probability density for a free particle at rest, without performing quantum jumps. The figures show the density at five equidistant time steps.





3. OUTLOOK

The model allows for a number of extensions. For instance, alternate forms of the C_m operators allow for treatment of tunneling between detectors due to nonzero overlap of their eigenfunctions, as well as simulation of multi-particle systems.

Substituting the simple detectors with two-level systems would allow for a model of a weakly-measured particle: For sufficient overlap with the particle, the detector would be excited into a higher state, relaxation back to the ground state giving the "detection signal" without destroying the total wavefunction.

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References:

[1] Klaus Mølmer, Yvan Castin, and Jean Dalibard, Monte Carlo wave-function method in quantum optics, *J. Opt. Soc. Am. B* **12**, 4, pp. 524-538, 1993