PAULI CRYSTALS

Mariusz Gajda, Filip Gampel, Jan Mostowski, Tomasz Sowiński, Magdalena Załuska-Kotur

Institute of Physics, Polish Academy of Sciences

Pauli Crystals are groups of spin-polarized fermionic atoms in a trap. The atoms **do not interact**, but due to Fermi statistics, there are **correlations** between their positions. Simultaneously measuring these positions reveals the correlations and **unexpected geometric structures** arise solely from the Pauli exclusion principle.

OVERVIEW

Consider a group of fermionic atoms (e.g. Li-6) with polarized spin in a harmonic trap. The atoms are **at low temperature** – all atomic movement in the trap is frozen, and the atoms are in the harmonic oscillator ground state.

The atoms **do not interact**, but due to Fermi statistics, there are **correlations** between their positions: predicted in 2016 [1] and observed in 2020 [2].

One particle wavefunction: $\phi_j(x_k)$

Many-body wavefunction: $\Psi(x_1, ..., x_N) = \frac{1}{\sqrt{N!}} \mathrm{Det}\{\phi_j(x_i)\}$

Atomic positions are random, and their distribution – the square modulus of the many-body wavefunction – is a function of N_d variables (N = atom number, d = number of dimensions). **This is tough to visualize and study.**

One particle density function: The probability density of finding an atom at position x without taking into account the positions of the remaining atoms.

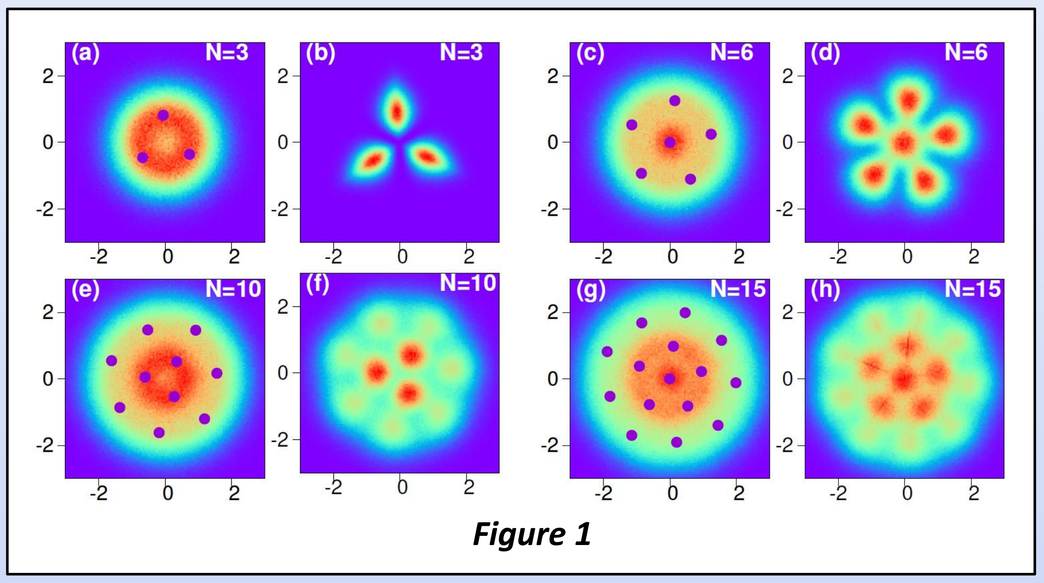

Figure 1

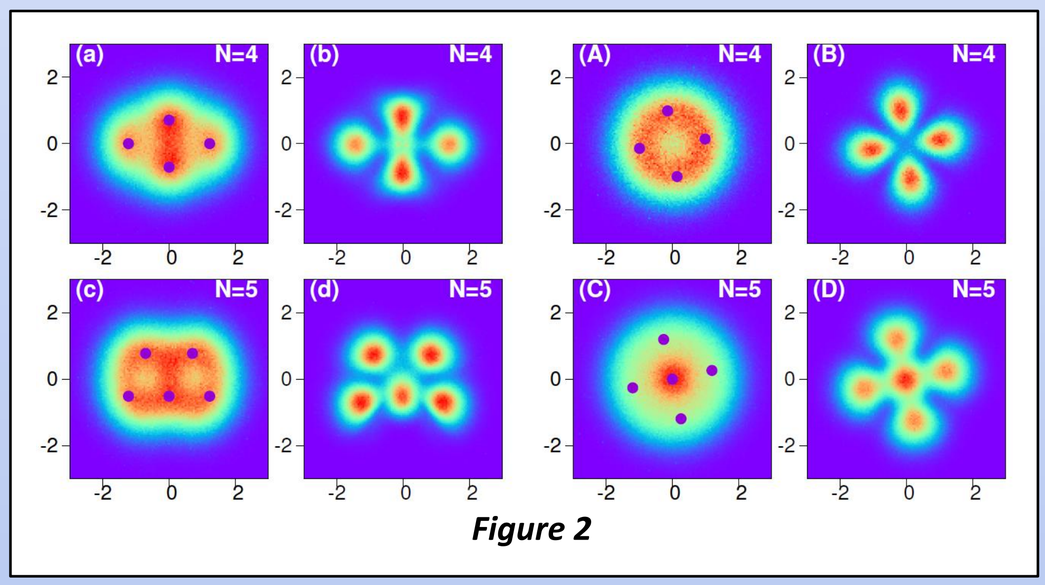

Figure 2

TAKING CORRELATIONS INTO ACCOUNT

Let us first assume three atoms in a two-dimensional trap (N=3, d=2). The atoms are in the ground state – one atom in the lowest energetic level, two in the first excited level, one in x, the other in y-direction.

The one particle density function can be seen in Fig. 1, upper left. The most probable configuration according to the many-body distribution is marked with black dots. The maxima form an **equilateral triangle**.

We expect that most of the time, the particles will be found close to the maxima of the many-body distribution. Using a Monte Carlo method, we compute an ensemble of atomic configurations, simulating a series of real measurements. The positions form a triangle, usually similar to an equilateral one, but each time rotated by a random angle.

To compare and quantify them, we develop a method **of recognizing patterns and re-aligning the individual outcomes**.

Other cases corresponding to closed energy shells are N=6, N=10, N=15 (see Fig. 1). Pauli crystals for N=10 and N=15 have a different geometry then crystals built by interacting (repulsive) particles in a trap, for instance ions.

The Pauli exclusion principle may not be treated as a form of particle interaction.

In the case of **open shells** there is degeneracy – there are different shapes of Pauli crystals corresponding to equal energies. Fig. 2 shows such Pauli crystals for N=4 and 5, d=2.

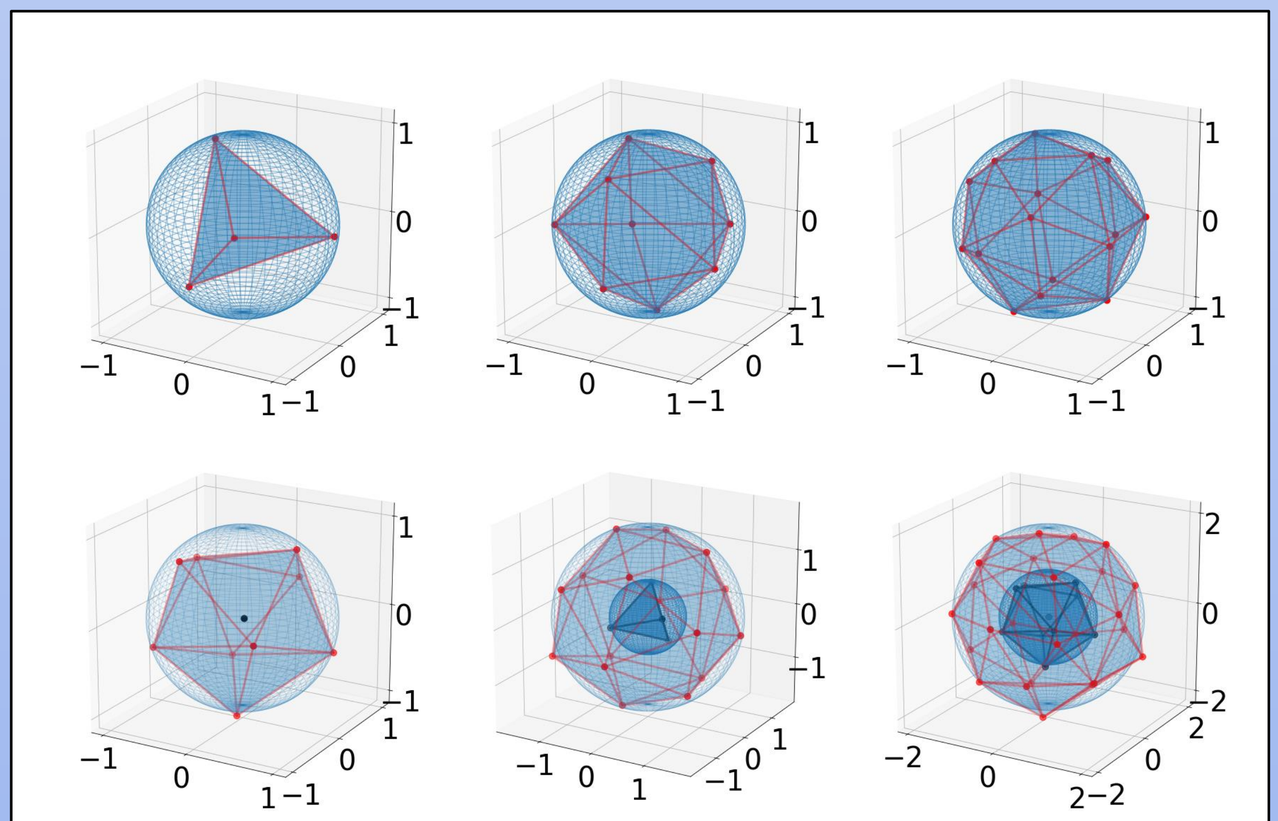

***Figure 3**: Pauli Crystals in three dimensions. **The upper row** shows N=4, 9 and 16 free particles located on a sphere (red vertices, connected by edges for improved readability). Note that unlike in the other cases, the particles do not form geometrically seperate shells. **The lower row** shows N=10, 20 and 35 particles in a three-dimensional harmonic trap. The particles form spatial shells, visualised by spheres of different sizes.*

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References:

[1] Mariusz Gajda, Jan Mostowski, Tomasz Sowiński, Magdalena Załuska-Kotur, Single-shot imaging of trapped Fermi gas, *EPL* **115**, 20012, 2016

[2] Marin Holten, Luca Bayha, Keerthan Subramanian, Carl Heintze, Philipp M Preiss, Selim Jochim, Observation of Pauli Crystals, *PRL* **126**, 020401, 2021

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