Entanglement dynamics of NV centers coupled to a bath of nuclear spins

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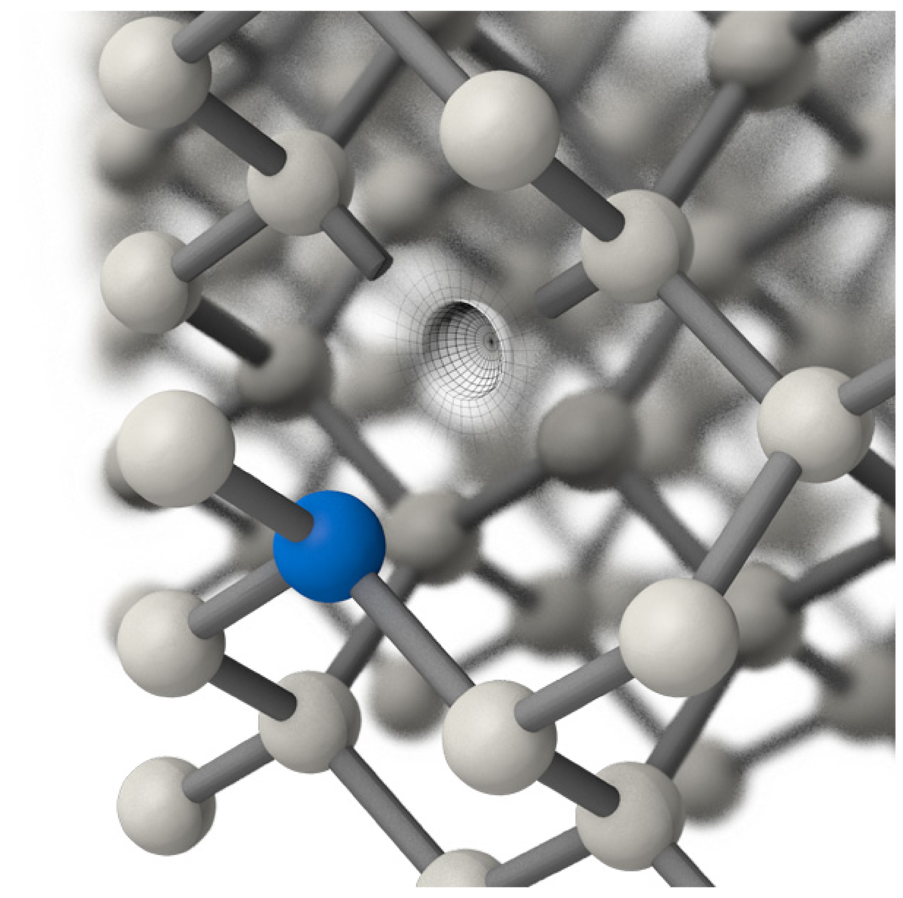
Nitrogen-Vacancy Center



- The nitrogen-vacancy center [1,2] is a complex of a nitrogen atom with a neighboring vacancy in diamond lattice.
- Ground state manifold corresponds to spin $S = 1$.
- Zero-field splitting of 2.88 GHz between $m_s = 0$ and $m_s = \pm 1$ state.
- Efficient optical initialization of $m_s = 0$
- Quantization axis $\hat{\mathbf{z}}$ is set along the direction of nitrogen-vacancy
- Echo coherence times $\sim 500\ \mu$s for natural diamond, longer in isotopically purified samples.
- Entanglement of two nearby centers was deterministically created [3].

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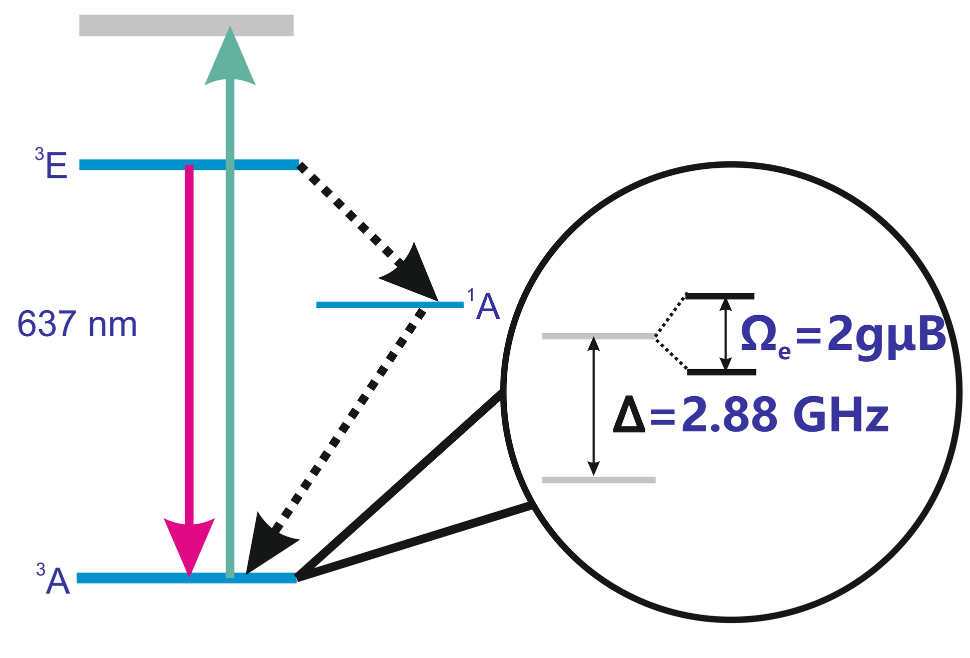


NV center energy levels.

Nuclear bath in diamond and its Hamiltonian

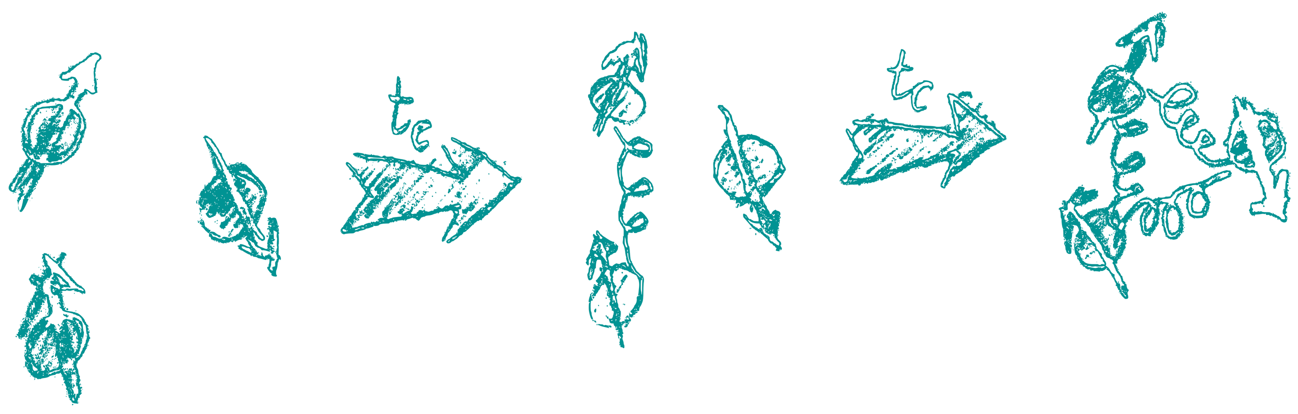
In natural diamond 1.1% of carbon atoms are spin-1/2 ^{13}C isotopes. These nuclear spins interact with the NV center qubit by hyperfine dipolar interaction, and their influence dominates the dephasing of the qubit. The Hamiltonian is:

$$\mathscr{H} = \mathscr{H}_{NV} + \mathscr{H}_{bath} + \mathscr{H}_{int}\,, \qquad (1)$$

where $\mathscr{H}_{NV} = \Delta (S^z)^2 + \frac{\Omega_e}{2} S^z$ with Ω_e being the NV Zeeman splitting, the bath Hamiltonian consists of Zeeman and dipolar interaction terms:

$$\mathscr{H}_{bath} = \frac{\omega_k}{2} \sum_k I^z_k + \sum_{k \neq l} b_{kl} (I^+_l I^-_l + I^-_k I^+_l - 4 I^z_k I^z_l)\,, \qquad (2)$$

and the interaction term is due to dipolar center-nuclear couplings: $\mathscr{H}_{int} = \sum_k A_k S_z I^z_k$ (we neglect here anisotropic $S_z I^{x,y}$ couplings, as they are irrelevant at considered magnetic fields).



Decoherence of a single qubit (for free evolution, echo, or dynamical decoupling experiments) can be calculated with high accuracy with **Cluster-Correlation Expansion (CCE)** method [4,5]:

- Contributions to decoherence from non-trivial single-, two-, three- and higher-spin correlations are calculated.
- Convergence is due to the fact that nontrivial multi-spin correlations do not have the time to build up on timescale on which qubit's coherence is already significantly diminished.
- For echo decay it is enough to stop at CCE-2, i.e. calculate the dynamics of **nuclear pairs**.
- **Strongly coupled** pairs have $A_k - A_l \gg b_{kl}$. They can be found close to the qubit, and they leave oscillatory fingerprints in echo signal.
- **Weakly coupled** pairs give the decay envelope.

Entanglement decay due to pure dephasing

We focus on 4 Bell states of the entangled qubits, i.e.:

$$\begin{cases} |\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle) \\ |\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \end{cases}$$

In order to use a pure-dephasing approximation for dynamics of Bell states, one needs to **neglect the dipolar flip-flop** between the NV centers by introducing a large enough magnetic field gradient, e.g. $G \approx 10^{-4}$ [T/nm] from a nearby nanomagnet [6], which sets a condition on the distance between NV centers to be: $\mathbf{d \gg 2}$ **nm**.

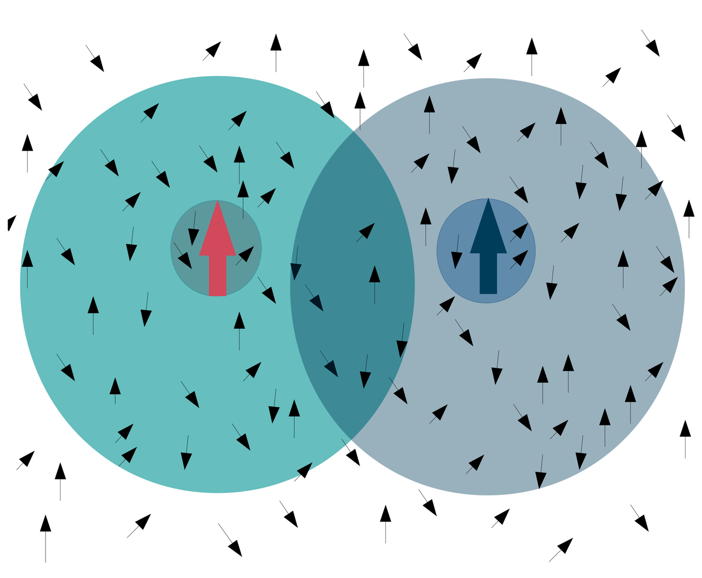
In such situation, the **measure of entanglement** (e.g. concurrence) is proportional to **the off-diagonal component** of the 2-qubit density matrix [7], denoted here by $W_{\Psi/\Phi}$.

- For completely common bath, $W_\Psi = 1$. This happens for $d < 1$ nm.
- Partially common bath gives $W_\Psi > W_\Phi$ when the noises experienced by the qubits are **correlated**, and $W_\Psi < W_\Phi$ we they are **anticorrelated**.
- Convenient to measure the common bath effects with

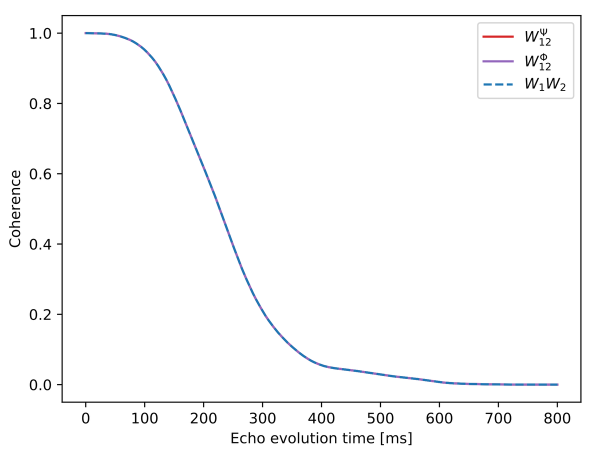
$$\Lambda_{\Psi/\Phi}(t) = \frac{W_{\Psi/\Phi}(t)}{W_1(t) W_2(t)} \qquad (3)$$

- For Gaussian noise $W_{\Psi/\Phi} = W_1 W_2 \exp(\mp \chi_{12})$ so that $\Lambda_\Phi = \Lambda_\Psi^{-1}$.

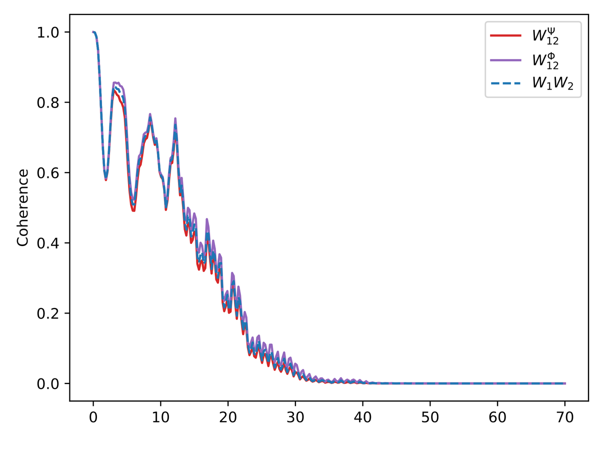
Common bath for 2 entangled NV centers



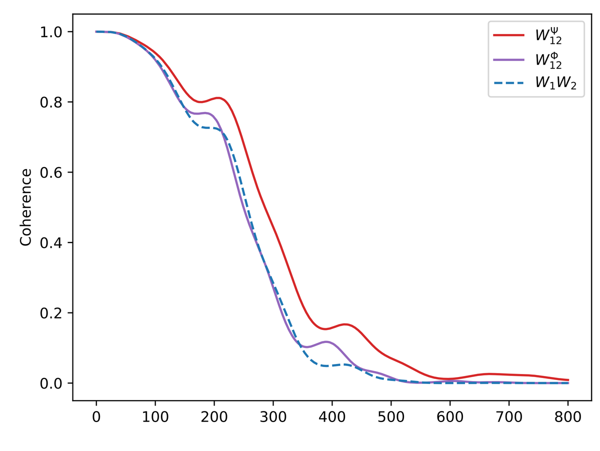
Common bath for 2 NV centers. Pink and blue arrow correspond to 2 NV centers in the system, small arrows represent ^{13}C nuclei forming a bath and the coloured areas show the environments dominating the decoherence of each qubit. Close to the NV center qubits, there exists a part of environment, where nuclear dimers are leaving strong fingerprints in single qubit decoherence [8]. There probabilty of having strongly coupled pairs close to the center is about 50%.



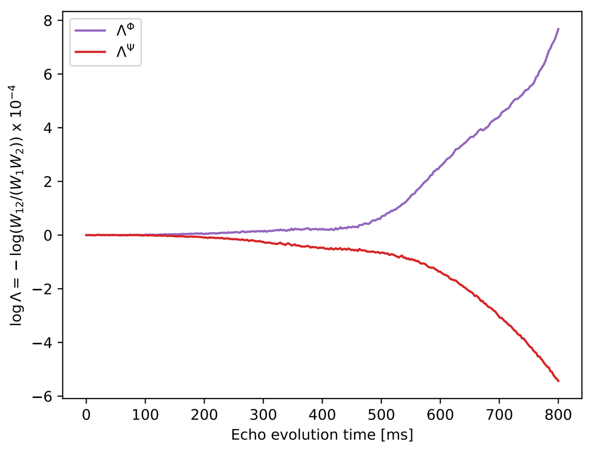
Echo signal for ^{13}C concentration of 0.01 % and interqubit distance $d = 200$ nm - the case of almost separate baths and uncorrelated decoherence.



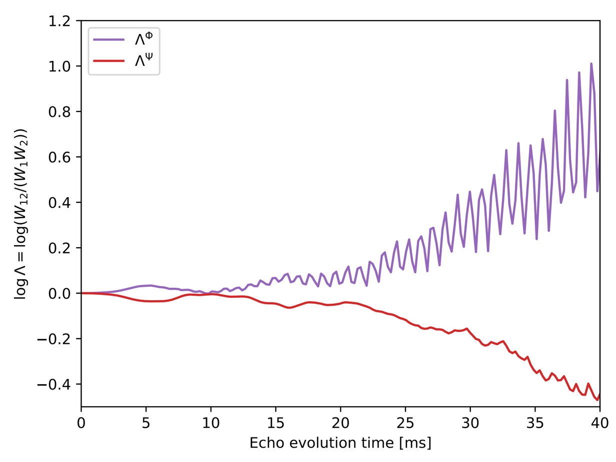
Echo signal for 0.1 % of ^{13}C and $d = 10$ nm. Effects of correlated baths are beginning to be visible.



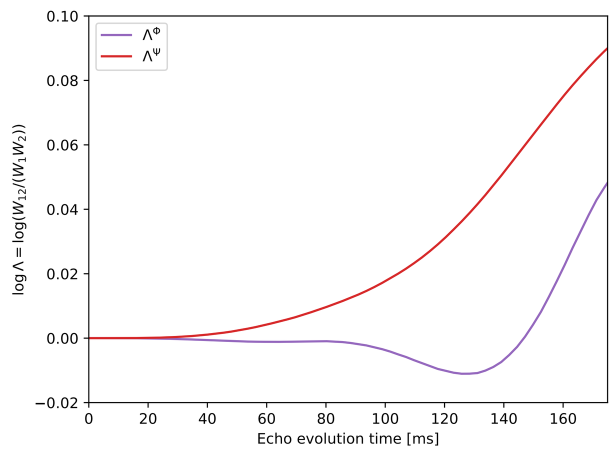
Echo signal for 0.01 % of ^{13}C and $d = 10$ nm. Common bath regime is approached, as $W_\Psi > W_\Phi$.



$\Lambda(t)$ corresponding to results from the left panel. $\log \Lambda_\Phi \approx -\log \Lambda_\Psi$ as for classical Gaussian noise due to decoherence being dominated by weakly coupled pairs.



$\Lambda(t)$ corresponding to results from the left panel. $\log \Lambda_\Phi \approx -\log \Lambda_\Psi$ still holds.



$\Lambda(t)$ corresponding to results from the left panel. Relations for Gaussian noise are clearly broken.

Acknowledgments: This research is supported by funds of Polish National Science Center (NCN), grants no. DEC-2012/07/B/ST3/03616 and DEC-2015/19/B/ST3/03152.

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