Missing level statistics and power spectrum analysis of three-dimensional chaotic microwave cavities

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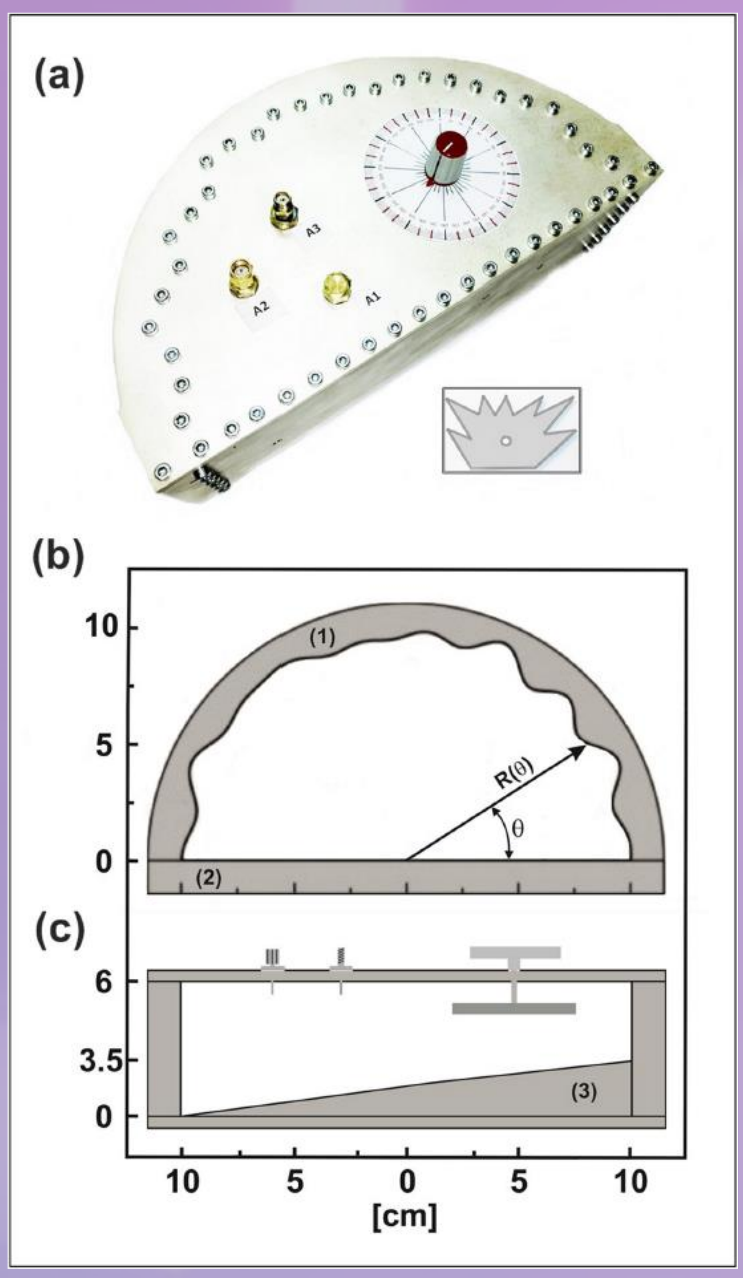
INTRODUCTION

We present the experimental study of missing level statistics [1] of three-dimensional chaotic microwave cavities. The investigation is reinforced by the power spectrum analysis which also takes into account missing levels. On the basis of our data sets we demonstrate that the power spectrum of levels fluctuations in combination with short- and long-range spectral fluctuations provides a powerful tool for the determination of the fraction of missing levels in systems that display wave chaos such as the three-dimensional chaotic microwave cavities. The experimental results are in good agreement with the theoretical expressions.

EXPERIMENTAL SETUP



Panel (a): A photograph of the 3D microwave cavity showing the antenna positions A1, A2 and A3. The inset shows the scatterer which was rotated to obtain different realizations of the 3D cavity.

Panels (b) and (c) show the sketch of the cavity on the cross-section plane and the position of the scatterer inside the cavity.

Balian-Bloch formula

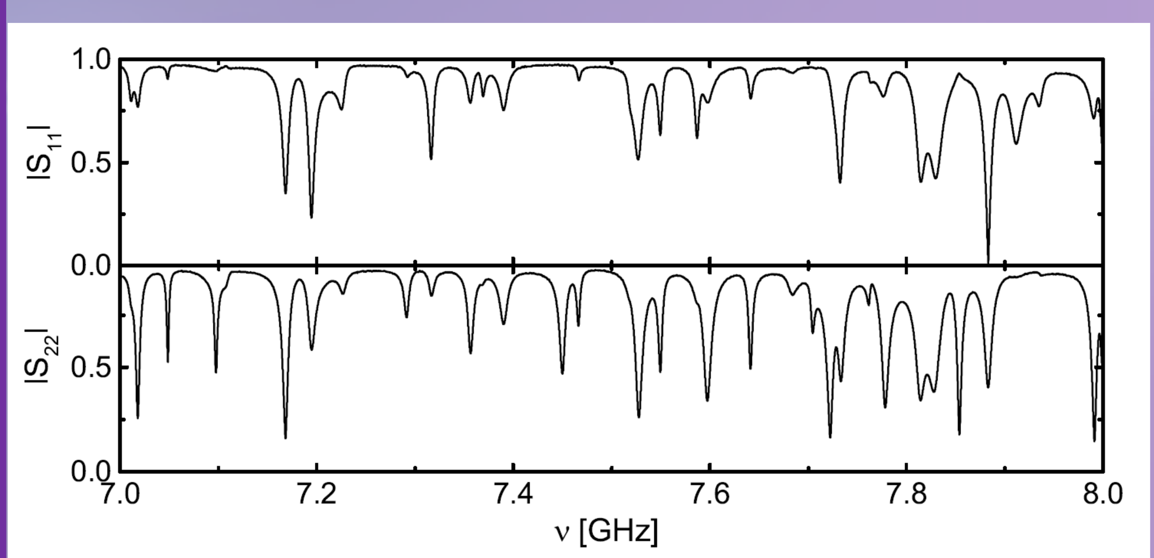
$$N(k) = \frac{1}{3\pi^2} V k^3 - \frac{2}{3\pi^2} k \int_s d\sigma_\omega / R_\omega$$

FLUCTUATIONS IN THE EXPERIMENTAL SPECTRA

Commonly used measure for short-range spectral fluctuations is the nearest-neighbor spacing distribution (NNSD), that is, the distribution of the spacings between adjacent eigenvalues, $s_i = \epsilon_{i+1} - \epsilon_i$ or its integrated form I(s). For long-range spectral fluctuations we considered the spectral rigidity of the spectrum $\Delta_3(L)$, given by the least-squares deviation of the integrated resonance density of the eigenvalues from the straight line best fitting it in the interval L [2] and the power spectrum of the deviation of the qth nearest-neighbor spacing from its mean value q, $\eta_q = \epsilon_{q+1} - \epsilon_1 - q$. The power spectrum is given in terms of the Fourier spectrum from "time" q to k, $S(k) = |\tilde{\eta}_k|^2$ with

$$\tilde{\eta}_k = \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} \eta_k \, exp\left(-\frac{2\pi i k q}{N}\right)$$

when considering a sequence of N levels



The measured modules of the elements $|S_{11}|$ and $|S_{22}|$ of the two-port scattering matrix S (ports A1 and A3) of the 3D microwave cavity in the frequency range 7− 8 GHz.

MISSING LEVEL STATISTICS

The fraction of detected resonances is characterized by the parameter φ, where 0 < φ ≤ 1.

The nearest-neighbor spacing distribution p(s)

$$p(s) = \sum_{n=0}^{\infty} (1-\varphi)^n P\left(n, \frac{s}{\varphi}\right)$$

For the GEO systems terms of spacing distribution are approximated by

First term

$$P\left(0, \frac{s}{\varphi}\right) = \frac{\pi}{2} \frac{s}{\varphi} exp\left[-\frac{\pi}{4}\left(\frac{s}{\varphi}\right)^2\right]$$

Second term

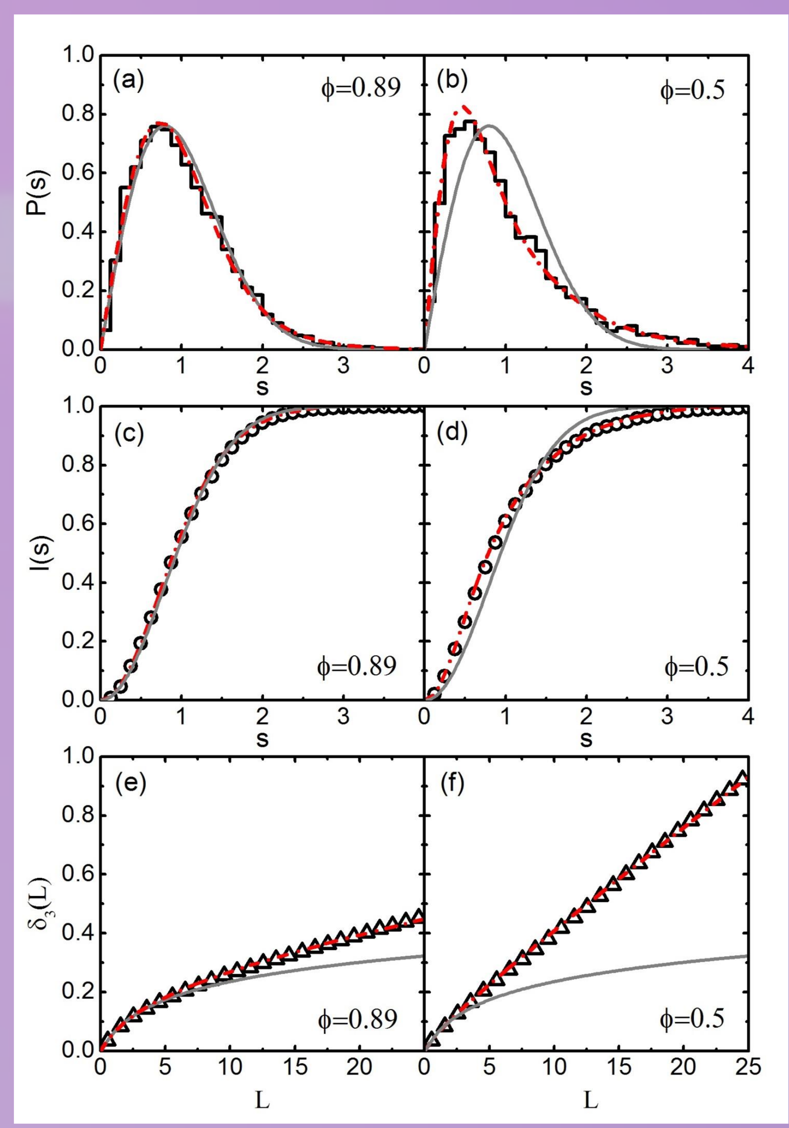
$$P\left(1, \frac{s}{\varphi}\right) = \frac{8}{3\pi^3} \left(\frac{4}{3}\right)^5 \left(\frac{s}{\varphi}\right)^4 exp\left[-\frac{16}{9\pi}\left(\frac{s}{\varphi}\right)^2\right]$$

Higher spacing distributions (n≥2), Gaussian asymptotic forms

$$P\left(n, \frac{s}{\varphi}\right) = \frac{1}{\sqrt{2\pi V^2(n)}} exp\left[-\frac{\left(\frac{s}{\varphi} - n - 1\right)^2}{2V^2(n)}\right]$$

Variances

$$V^2(n) \simeq \Sigma^2(L = n) - \frac{1}{6}$$



The integrated NNSD

$$I(s) = \int_0^s p(s')ds'$$

The spectral rigidity $\delta_3(L)$ [3]

$$\delta_3(L) = (1-\varphi)\frac{L}{15} + \varphi^2 \Delta_3\left(\frac{L}{\varphi}\right)$$

Panel (a) and (b): NNSD (histogram) ***Panel (c) and (d)***: Integrated NNSD (circles). ***Panel (e) and (f)***: The spectral rigidity of the spectrum (triangles). The experimental results are compared to those of the eigenvalues of random matrices from the GOE (grey full lines) for φ = 1.

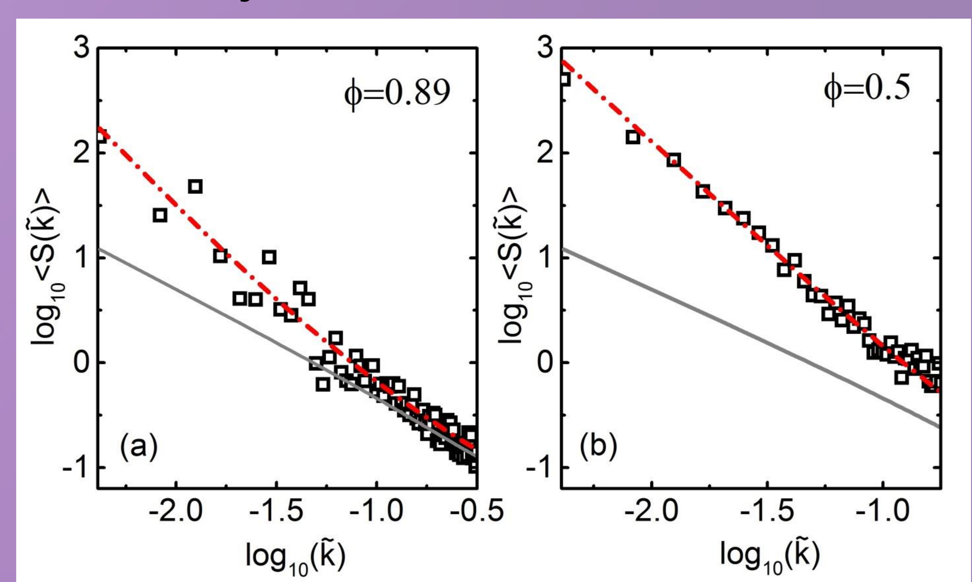
The corresponding missing-level statistics (red broken lines) were calculated for the GOE system for φ = 0.89 ***(panels (a) (c) and (e))*** and for φ = 0.5 ***(panels (b) (d) and (f))***.

POWER SPECTRUM

The power spectrum formula for incomplete spectra is given in Ref. [4].

$$\langle s(\tilde{k}) \rangle = \frac{\varphi}{4\pi^2}\left[\frac{K(\varphi\tilde{k}) - 1}{\tilde{k}^2} + \frac{K\left(\varphi(1-\tilde{k})\right) - 1}{(1-\tilde{k})^2}\right] + \frac{1}{4 sin^2(\pi\tilde{k})} - \frac{\varphi^2}{12}$$

Here, $0 \le \tilde{k} \le 1$ and $K(\tau)$ is the spectral form factor, which equals $K(\tau) = 2\tau - \tau \log(1 + 2\tau)$ for the GOE systems.



Panel (a) and (b): The average power spectrum (diamonds). The experimental results are compared to those of the eigenvalues of random matrices from the GOE (grey full lines). The missing-level statistics (red broken lines) were calculated for the GOE system for φ=0.89 ***panel (a)*** and for φ=0.5 ***panel (b)***.

CONCLUSSIONS The experimental results are in good agreement with the theoretical predictions for missing level statistics, and for the power spectrum. All these expressions explicitly take into account the fraction of observed levels φ. The spectral rigidity of the spectrum and particularly the power spectrum appeared to be very sensitive to it.

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