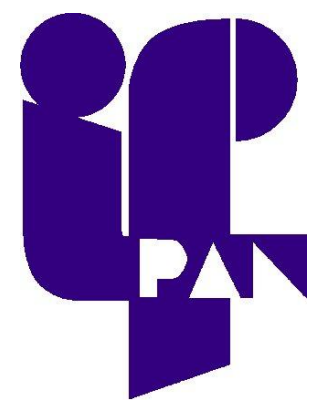


# Interactive optomechanical coupling with nonlinear polaritonic systems



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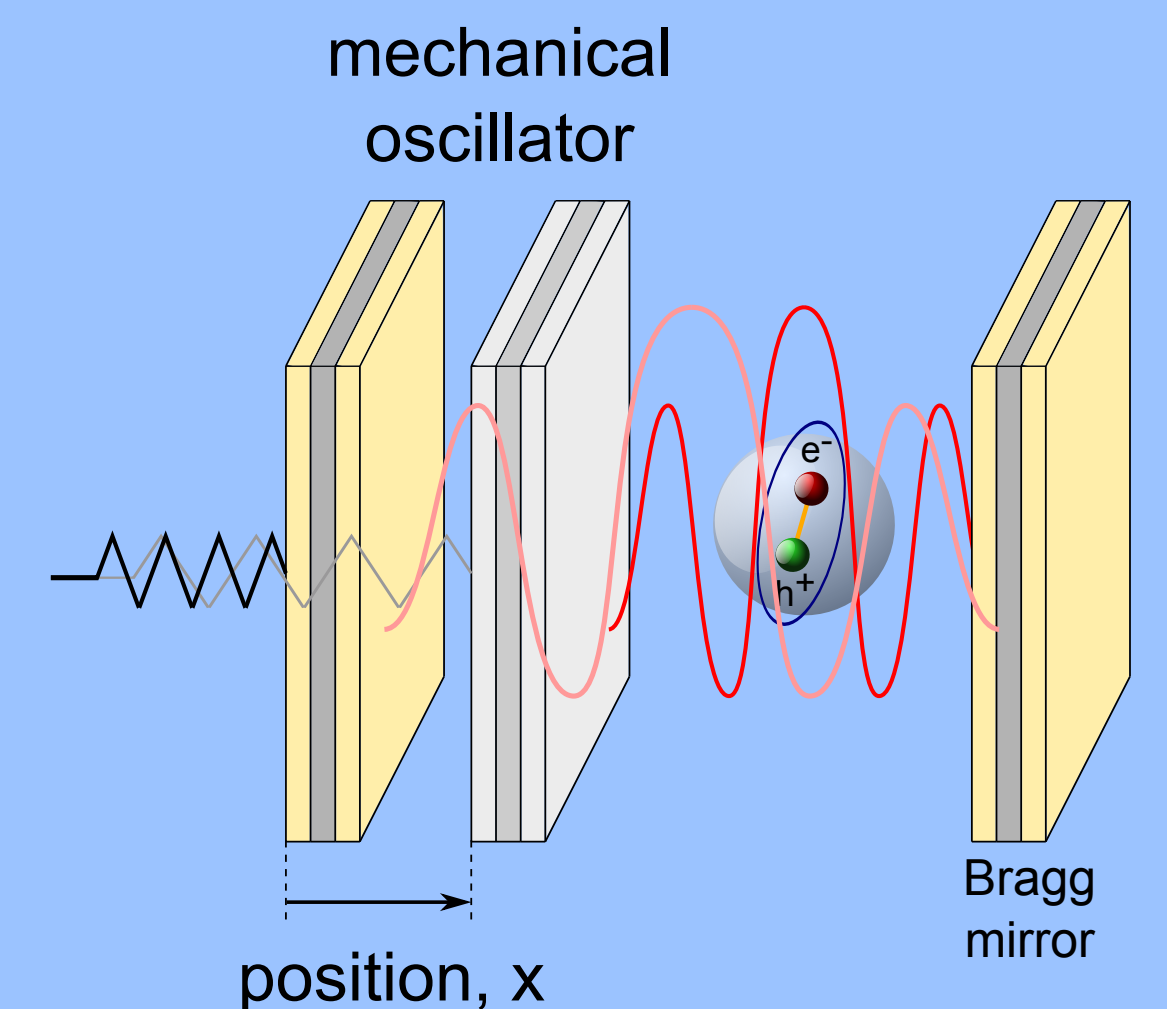
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## Abstract

We study a system of interacting matter quasiparticles strongly coupled to photons inside an optomechanical cavity. The resulting normal modes of the system are represented by hybrid polaritonic quasiparticles, which acquire effective nonlinearity. Its strength is influenced by the presence of the mechanical mode and depends on the resonance frequency of the cavity. This leads to an **interactive** type of optomechanical coupling, being distinct from the previously studied dispersive and dissipative couplings in optomechanical systems. The emergent interactive coupling is shown to generate effective optical nonlinearity terms of high order, being quartic in the polariton number. We consider particular systems of exciton-polaritons and dipolaritons, and show that the induced effective optical nonlinearity due to the interactive coupling can exceed in magnitude the strength of Kerr nonlinear terms, such as those arising from polariton-polariton interactions. As applications, we show that the higher order terms give rise to localized bright flat top solitons, which may form spontaneously in polariton condensates.



## The model

Hamiltonian describing the optomechanical coupling in a dipolariton microcavity

$$\begin{aligned} \hat{\mathcal{H}}_{\text{dpl}} = & E_C(x) \hat{a}_C^\dagger \hat{a}_C + E_{\text{DX}} \hat{a}_{\text{DX}}^\dagger \hat{a}_{\text{DX}} + E_{\text{IX}} \hat{a}_{\text{IX}}^\dagger \hat{a}_{\text{IX}} + \hbar \omega_m \hat{b}^\dagger \hat{b} \\ & + \left( \frac{\Omega}{2} \hat{a}_{\text{DX}}^\dagger \hat{a}_C + \frac{J}{2} \hat{a}_{\text{IX}}^\dagger \hat{a}_{\text{DX}} + \text{h.c.} \right) + U_{\text{DX}} \hat{a}_{\text{DX}}^\dagger \hat{a}_{\text{DX}} \hat{a}_{\text{DX}} \hat{a}_{\text{DX}} \\ & + U_{\text{IX}} \hat{a}_{\text{IX}}^\dagger \hat{a}_{\text{IX}} \hat{a}_{\text{IX}} \hat{a}_{\text{IX}} + U_{\text{DI}} \hat{a}_{\text{DX}}^\dagger \hat{a}_{\text{IX}} \hat{a}_{\text{DX}} \hat{a}_{\text{IX}}, \end{aligned}$$

To describe the interactive optomechanical coupling in the dipolariton system, we switch to the diagonal basis and consider only the lower mode, with the Hamiltonian

$$\begin{aligned} \hat{\mathcal{H}}_L = & E_L(x) \hat{\psi}^\dagger \hat{\psi} + \hbar \omega_m \hat{b}^\dagger \hat{b} + [U_{\text{DX}} \beta_D(x)^4 \\ & + U_{\text{IX}} \beta_I(x)^4 + U_{\text{DI}} \beta_D(x)^2 \beta_I(x)^2] \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi}, \end{aligned}$$

the Hopfield factors representing the indirect and direct exciton fractions of the lower dipolariton state

x-dependent terms in Hamiltonian can be expanded about the point  $x = 0$

$$E_L(x) \approx E_L(0) + x \frac{\partial E_L}{\partial x} = E_L(0) - g_0 (\hat{b} + \hat{b}^\dagger) \frac{\partial E_L}{\partial E_C}$$

$$\beta_{\text{D,I}}(x)^4 \approx \beta_{\text{D,I}}(0)^4 - g_0 (\hat{b} + \hat{b}^\dagger) \frac{\partial \beta_{\text{D,I}}^4}{\partial E_C},$$

$$\beta_D(x)^2 \beta_I(x)^2 \approx \beta_D(0)^2 \beta_I(0)^2 - g_0 (\hat{b} + \hat{b}^\dagger) \frac{\partial (\beta_D^2 \beta_I^2)}{\partial E_C}.$$

quantum description of  $x$

$$\hat{x} = x_{\text{ZPF}} (\hat{b} + \hat{b}^\dagger)$$

constant

$$g_0 = -x_{\text{ZPF}} \partial E_C / \partial x$$

The polariton-mechanical Hamiltonian becomes:

$$\begin{aligned} \hat{\mathcal{H}}_L = & E_L(0) \hat{\psi}^\dagger \hat{\psi} + \hbar \omega_m \hat{b}^\dagger \hat{b} + [U_{\text{DX}} \beta_D(0)^4 + U_{\text{IX}} \beta_I(0)^4 \\ & + U_{\text{DI}} \beta_D(0)^2 \beta_I(0)^2] \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} + \hat{\zeta} (\hat{b} + \hat{b}^\dagger), \end{aligned}$$

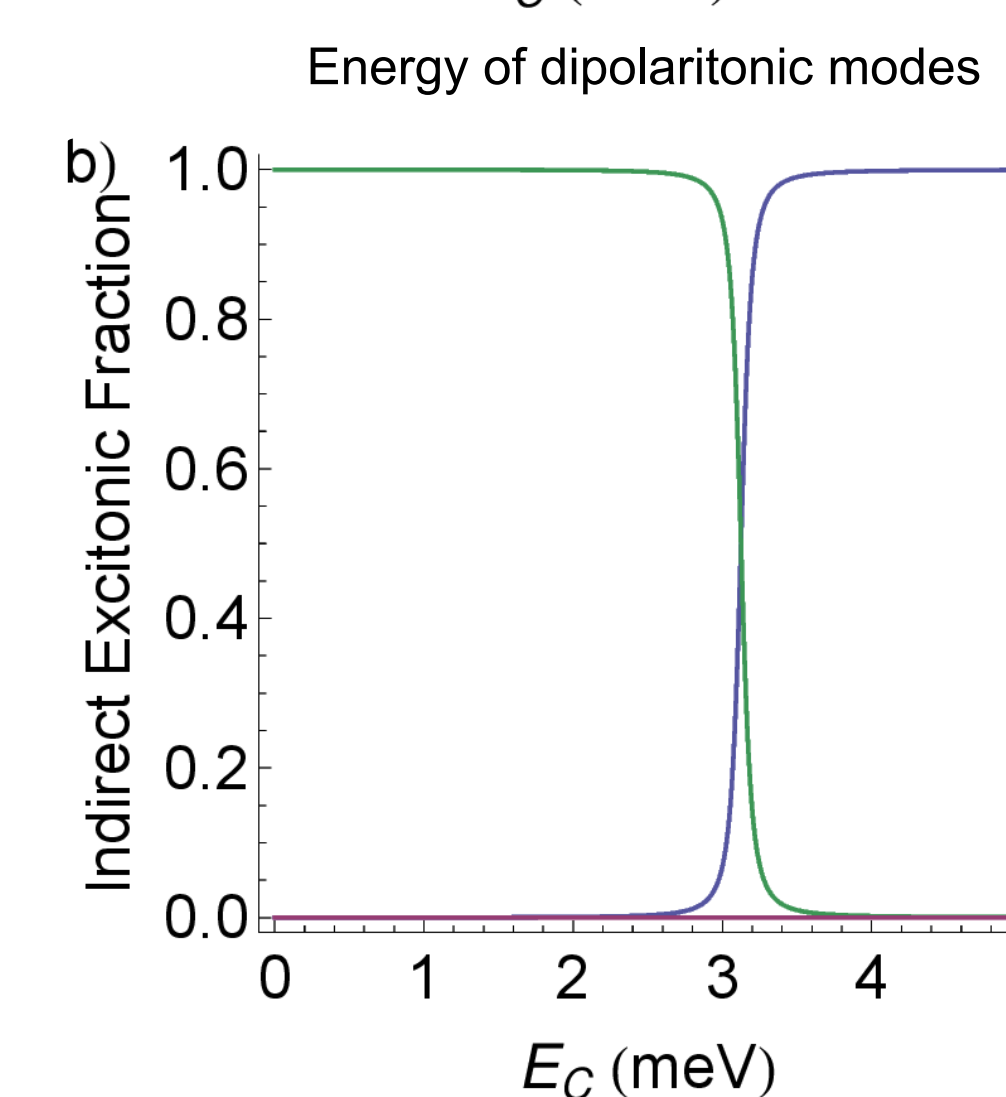
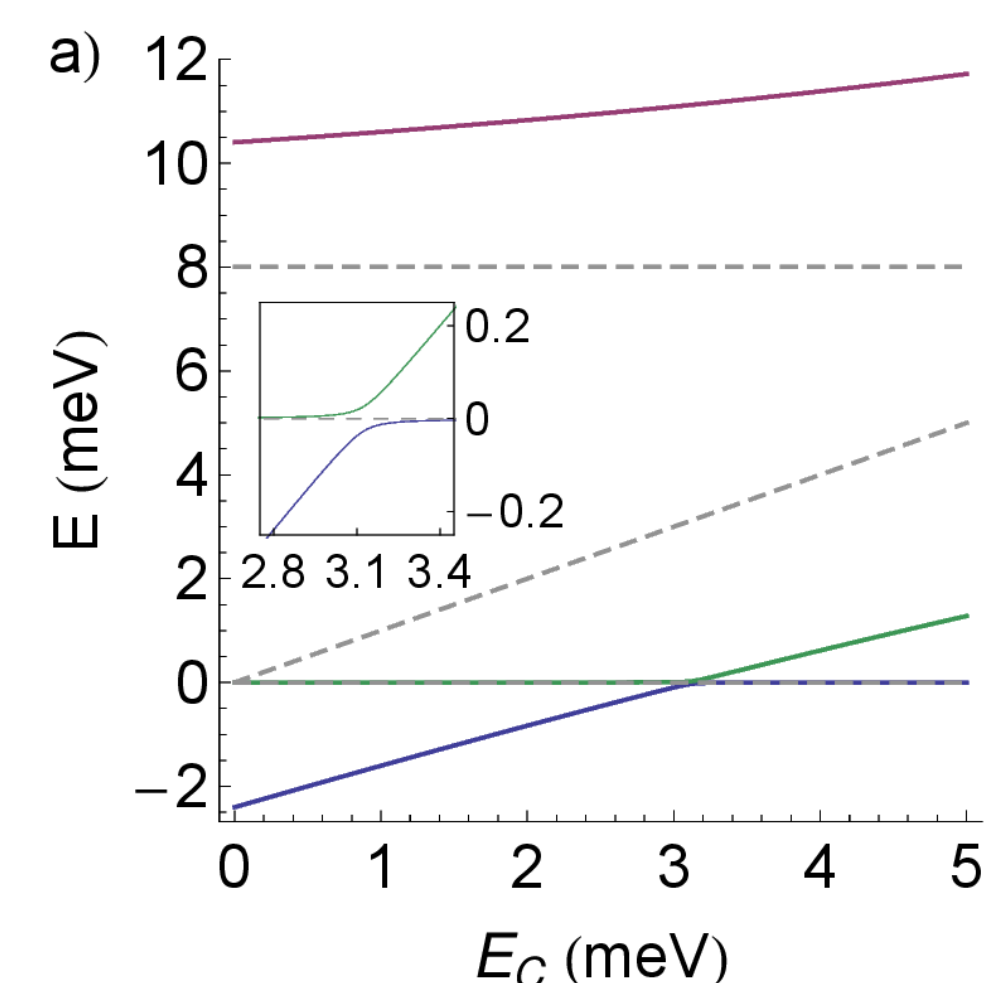
where

$$\begin{aligned} \hat{\zeta} = & -g_0 \left( U_{\text{DX}} \frac{\partial \beta_D^4}{\partial E_C} + U_{\text{DI}} \frac{\partial (\beta_D^2 \beta_I^2)}{\partial E_C} + U_{\text{IX}} \frac{\partial \beta_I^4}{\partial E_C} \right) \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} \\ & - g_0 \frac{\partial E_L}{\partial E_C} \hat{\psi}^\dagger \hat{\psi}. \end{aligned}$$

is the interactive coupling for the dipolariton mode

The polaron (Schrieffer-Wolff) transformation eliminates the phonon modes and the transformed Hamiltonian can be rewritten using the Baker-Campbell-Hausdorff formula:

$$\begin{aligned} \hat{\mathcal{H}}'_L = & E_L(0) \hat{\psi}^\dagger \hat{\psi} + \hbar \omega_m \hat{b}^\dagger \hat{b} + [U_{\text{DX}} \beta_D(0)^4 + U_{\text{IX}} \beta_I(0)^4 \\ & + U_{\text{DI}} \beta_D(0)^2 \beta_I(0)^2] \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi} - \frac{\hat{\zeta}^2}{\hbar \omega_m}, \end{aligned}$$



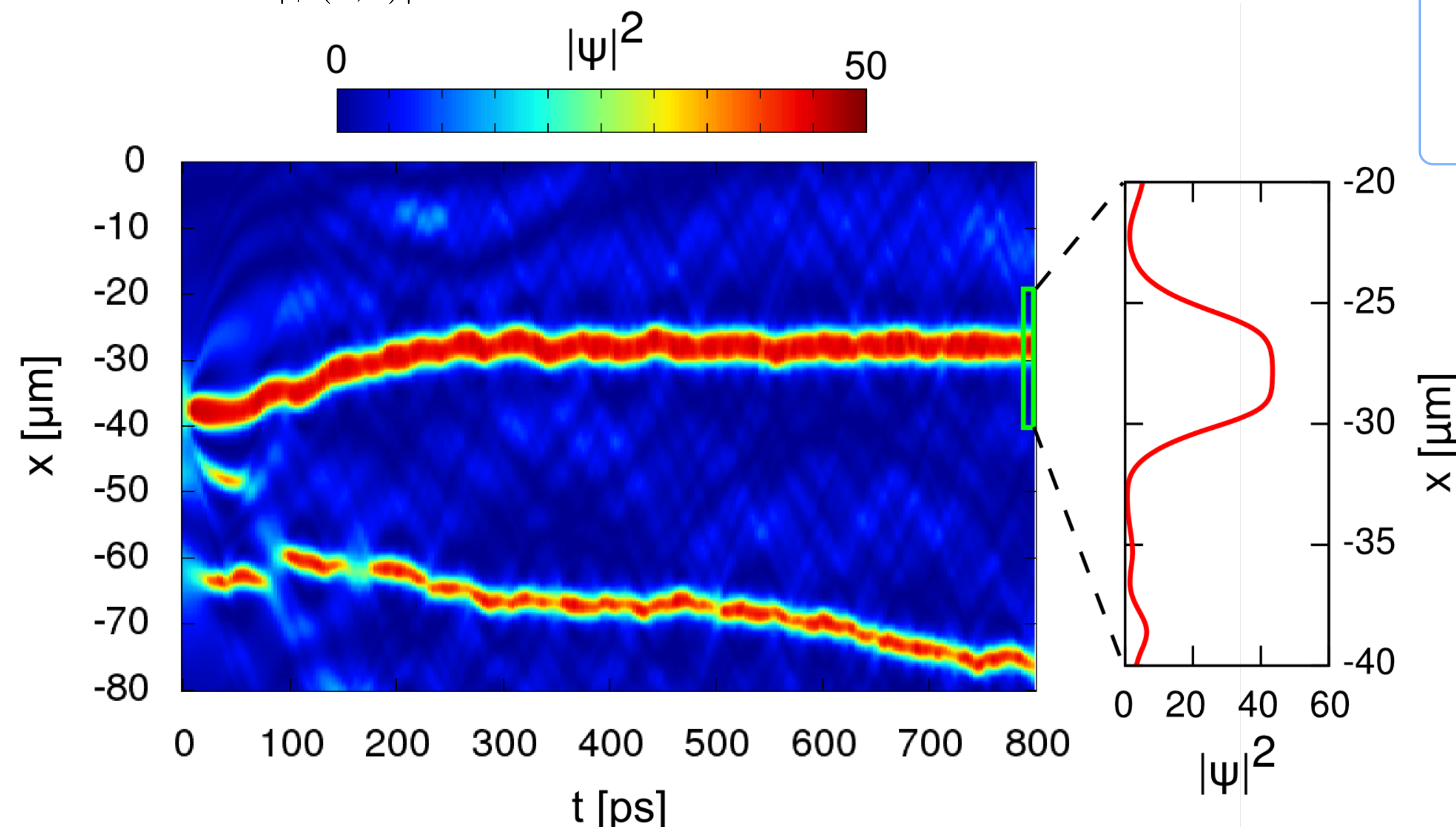
Indirect exciton fraction (Hopfield coefficient) for the three polariton modes

## The spatial dynamics of a polaritonic system

Gross-Pitaevskii-type equation for 1D

$$i\hbar \frac{\partial \psi(z, t)}{\partial t} = \left[ -\frac{\hbar^2 \nabla^2}{2m_{\text{LP}}} + \left( \alpha_{1\text{D}} - \frac{\gamma_{1\text{D}}^2}{\hbar \omega_m} \right) n - \frac{2\beta_{1\text{D}} \gamma_{1\text{D}} n^2 + \beta_{1\text{D}}^2 n^3}{\hbar \omega_m} + \delta n^4 \right] \psi(z, t)$$

where  $n = |\psi(z, t)|^2$



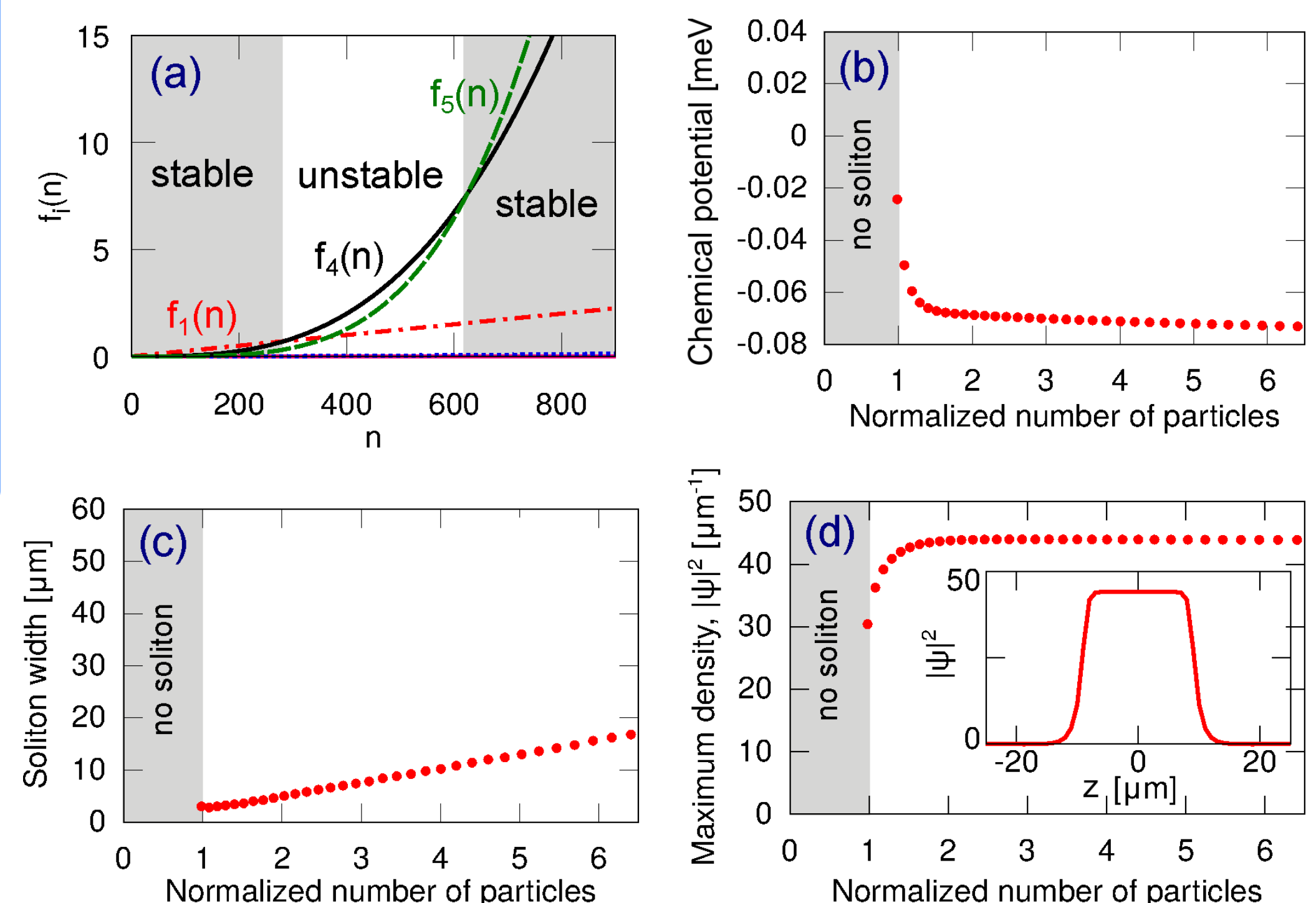
Stable flat-top solitons, emerging from an initial random noise, obtained by propagating of Gross-Pitaevskii type equation in real time.

Nonlinear interaction parameters now have dimensions consistent with a 1D polariton density  $n$ , and are related to previously described parameters by

$$\alpha_{1\text{D}} \approx U_{\text{IX}} A \beta_I(0)^4 / d$$

$$\beta_{1\text{D}} \approx g_0 U_{\text{IX}} / J(A/d)^{3/2}$$

$$\gamma_{1\text{D}} = g_0 (A/d)^{1/2} \partial E_L / \partial E_C \approx g_0 (A/d)^{1/2} / 2$$



(a) Comparison of the term amplitudes containing  $n$ . The other panels show the dependence of the chemical potential (b), soliton width (c), and soliton height (d) with respect to normalized number of particles  $N/N_0$  where  $N_0 = 119$ . The inset in panel (d) shows the flat-top soliton profile at  $N/N_0 = 6$ . Parameters:  $\alpha_{1\text{D}} = 0.005 \text{ meV}\mu\text{m}$ ,  $\beta_{1\text{D}} = 3.54 \times 10^{-4} \text{ meV}\mu\text{m}^{3/2}$ ,  $\gamma_{1\text{D}} = 4.42 \times 10^{-4} \text{ meV}\mu\text{m}^{1/2}$ ,  $\delta = 0.5 \times 10^{-6} \text{ meV}\mu\text{m}^4$ ;  $m_{\text{LP}}$  was taken as  $1.5 \times 10^{-4}$  of the free electron mass.

## In conclusion

We have introduced the notion of interactive optomechanical coupling, which appears in generic systems where the strength of Kerr nonlinearity is influenced by a mechanical motion. It has a highly nonlinear character, and manifests itself as a seventh order nonlinear susceptibility. As particular examples of systems where interactive coupling can be realized we describe exciton-polaritons and dipolaritons. In the latter case the steep dependence of Hopfield coefficients allows to attain large interactive coupling constant. The theory was applied to 1D polaritonic waveguides, revealing the generation of bright flat-top solitons in polaritonic fluids.

## Acknowledgements



NB and MM acknowledge support from National Science Center, Poland grants DEC-2011/01/D/ST3/00482 and 2015/17/B/ST3/02273.

TCHL was supported by the MOE AcRF Tier 1 grant 2016-T1-1-084 and MOE AcRF Tier 2 grant 2015-T2-1-055.

OK thanks I. A. Shelykh and A. S. Sørensen for the useful discussions, and acknowledges the support from the European Union Seventh Framework Programme through the ERC Grant QIOS (Grant No. 306576).

## References

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