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Localization of light in three-dimensional random dielectric media

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A systematic approach to the localization of light waves in three-dimensional dielectric media is developed. A general definition of localization of electromagnetic waves is proposed and its consequences are elaborated. A significant amount of localization of the energy density of the electromagnetic field is predicted in finite systems of randomly distributed dielectric particles modeled by dipoles linearly coupled to the electric field of the incident wave. Although in this case it is not possible to achieve perfect localization, the predicted phenomenon is experimentally indistinguishable from a complete localization. Our approach is based directly on the Maxwell equations; the vector character of the electromagnetic waves is fully taken into account. The concepts presented in our previous paper [M. Rusek and A. Orłowski, Phys. Rev. E **51**, R2763 (1995)] are now generalized to the three-dimensional case. Instead of using the Kirchhoff integral formula for scalar waves, we now analyze light scattering by pointlike dielectric particles as the special case of general considerations dealing with elastic scattering of electromagnetic waves by arbitrary localized charges and currents.

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I. INTRODUCTION

Investigations of the electron transport in disordered solids, usually semiconductors, led to the concept of localization of the electron wave functions [1]. This phenomenon, known now as the Anderson localization, became a prominent part of contemporary condensed matter physics and is still a vivid subject of theoretical and experimental research. In such disordered media the propagation of electrons is altered by the presence of a random potential. The Anderson localization is completely based on the interference effects in multiple elastic scattering. It is obvious, however, that interference is a common property of all wave phenomena. It is no wonder, therefore, that many generalizations of electron localization to other matter waves (neutrons) as well as classical waves (electromagnetic and acoustic waves) have been proposed [2–6]. Of course, apart from remarkable similarities, there are also striking differences. Very different is, e.g., the long-wavelength limit of elastic scattering. For electrons we have mainly s-wave scattering, which is spatially isotropic and wavelength independent. For light we observe p-wave scattering. In this case there is forward-backward symmetry but scattering is nonisotropic. In addition, in the long-wavelength limit, the cross section for scattering of electromagnetic waves shows the well-known λ^{-4} dependence. In inelastic scattering electrons change their energy but their total number is conserved. For light we have strong absorption and the intensity decreases. Moreover, electrons are described by scalar wave functions (or two-component spinors if the spin is included). To describe correctly localization of electromagnetic waves we need to consider, in general, three-dimensional vector fields.

The case of electromagnetic waves is of special interest as a variety of experimental investigations exist. One aspect of light localization should be emphasized, namely, the distinction between *weak localization* and *strong localization*. Weak localization is presently relatively well understood theoretically [7–9]. It manifests itself as enhanced coherent backscattering. This phenomenon is now experimentally established beyond any doubts. Striking coherent, back-directed peaks of intensity superimposed upon intensity of electromagnetic waves diffusely scattered from random media have been observed for different systems of randomly distributed scatterers forming three-dimensional [10–12] media. Since coherent backscattering affects the diffusion constant describing the propagation of electromagnetic waves in random media, the weak localization can be considered as the precursor of strong localization. The question as to whether interference effects in strongly scattering random media can reduce the diffusion constant to zero producing purely localized states depends on the dimension of the sample considered. It is commonly believed that, in analogy to the Anderson localization, any amount of disorder can produce localization of electromagnetic waves in one and two dimensions. In three dimensions a critical level of disorder must exist in order to localize light. From the experimental point of view there are indeed some reasonable indications that strong localization could be possible in three-dimensional random dielectric structures [13–17]. There is, however, no definite experimental confirmation. On the other hand, despite remarkable efforts no deeper insight into localization of light can be found in the literature. It concerns especially those problems where the polarization effects have to be taken into account. Such considerations should assume the vector character of electromagnetic fields from the very beginning. To achieve this goal in a consistent way they should be based directly on the Maxwell equations. On the other hand, they should be simple enough to provide calculations without too many too-crude approximations. In this paper we construct explicitly such a model for the three-dimensional localization of electromagnetic waves. The resulting model is thoroughly analyzed and its major consequences are elaborated.

A very common approach in investigations of the Anderson localization in the solid-state physics is to study a transport equation for the ensemble-averaged squared modulus of the wave function. Under some assumptions such a transport equation can be converted into a diffusion equation. Then the behavior of the diffusion constant is used to recognize localization: if the diffusion constant in the scattering medium becomes zero the (strong) localization is achieved. This approach has also been generalized to the electromagnetic waves. It has some advantages. The scaling theory of localization (investigating, among others things, the scale dependence of the diffusion constant or conductance) is well developed. Moreover, the diffusion constant, as well as other transport properties of waves in disordered samples, can be directly measurable. However, this standard approach does not emphasize the fundamental role played by the interference effects. In this paper we would like to propose and develop another theoretical approach based directly on the Maxwell equations. It ensures that all interference effects in elastic scattering are explicitly present. Also the vector character of the electromagnetic field is automatically taken into account. Instead of studying transport properties of monochromatic electromagnetic waves, we investigate the time-averaged energy density of the field in the strongly scattering random dielectric medium. The energy density is a very natural analog of the squared modulus of the quantum-mechanical wave function. Also from the experimental point of view this approach is completely justified. The recent experiment concerning localization of microwaves in the two-dimensional medium was based just on the measurements of the energy density in the medium [18]. In close analogy with quantum mechanics, we propose a remarkable definition of localization as well as quasilocalization of electromagnetic waves. Several aspects of such localization are then studied via a simple yet realistic model: we analyze light scattering by pointlike dielectric particles as the special case of general considerations dealing with elastic scattering of electromagnetic waves by arbitrary localized charges and currents. A significant amount of localization of the energy density of the electromagnetic field is predicted in systems of randomly distributed dielectric particles modeled by dipoles linearly coupled to the electric field of the incident wave. We prove that in finite media it is not possible to achieve perfect localization but the predicted phenomenon is practically indistinguishable from the complete localization. Although it is known that several mathematical problems emerge in the formulation of interaction of pointlike dielectric particles with electromagnetic waves [19–21], we show that it is possible to develop a consistent yet rather simple approach. This treatment may be viewed as a vector-field generalization of the results obtained in our previous paper [22], which were based on the Kirchhoff integral formula for scalar waves.

This paper is organized as follows. In Sec. II a definition of localized light waves in systems of dielectric particles, motivated by analogies with quantum mechanics as well as by the recent experiment concerning localization in two-dimensional random media, is proposed and its consequences are elaborated. Then we introduce the Maxwell equations in the integral form as a very convenient and effective tool for studying localization of light in such dielectric media. Using this formalism we investigate in Sec. III the general conditions imposed on the polarization of the lossless bounded medium, which should be satisfied to make localization of the energy density of the monochromatic light wave possible. It is proved that in systems consisting of a finite number of dielectric particles, localization of light is not possible. The results of these calculations are used later to consider localization as a special case of the interference effects in elastic scattering of light waves. Using the optical theorem, we show that the polarization of the general bounded and lossless medium is coupled to the electric field of the *incident* wave. From those fundamental considerations, we give in Sec. IV a simple derivation of the general form of the coupling between the pointlike three-dimensional dipole and the electric field of the light wave incident on it in the case of the elastic scattering. Section V is devoted to analysis of various aspects of light localization in systems consisting of randomly distributed dielectric particles modeled by *single* three-dimensional dipoles. The concept of quasilocalization is developed. The results of numerical investigations are presented and thoroughly discussed. Main results of our study are summarized in Sec. VI.

II. LOCALIZED WAVES

The standard approach to localization of monochromatic electromagnetic waves [2,23–29]

$$\vec{E}(\vec{r},t) = \mathrm{Re}\{\vec{\mathcal{E}}(\vec{r})e^{-i\omega t}\}, \tag{1a}$$

$$\vec{H}(\vec{r},t) = \mathrm{Re}\{\vec{\mathcal{H}}(\vec{r})e^{-i\omega t}\} \tag{1b}$$

is based on the similarities between the Helmholtz equation for the electric field amplitude in an isotropic lossless dielectric

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{\mathcal{E}}(\vec{r})] + k_0^2[1 - \epsilon(\vec{r})]\vec{\mathcal{E}}(\vec{r}) = k_0^2\,\vec{\mathcal{E}}(\vec{r}) \tag{2}$$

and the time-independent Schrödinger equation

$$\left\{-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})\right\}\psi(\vec{r}) = E\,\psi(\vec{r}). \tag{3}$$

The term $k_0^2[1-\epsilon(\vec{r})]$ corresponds to the potential $V(\vec{r})$ ensuring localization of the electron wave function and the squared wave number in vacuum $k_0^2 = \omega^2/c^2$ plays a role analogous to the energy eigenvalue E. It is well known, however, that there are several fundamental differences between quantum particles and electromagnetic waves [27]. In general, the electric field vector cannot be interpreted as the probability amplitude. The correct equivalent of the quantum-mechanical probability density is rather the energy density of the field and not the squared electric field. Therefore, after some introductory remarks we would like to propose a reasonable definition of localization of light based on the behavior of the energy density.

Usually localization of light is studied experimentally in microstructures consisting of dielectric spheres with diameters and mutual distances being comparable to the wavelength [29]. It is well known that the theory of multiple scattering of light by dielectric particles is tremendously simplified in the limit of point scatterers. In principle, this approximation is justified only when the size of the scattering particles is much smaller than the wavelength. In practical calculations, however, many multiple-scattering effects can be obtained qualitatively for coupled electrical dipoles. Examples are universal conductance fluctuations [30], enhanced backscattering [31], dependent scattering [19], and strong localization in two dimensions [22]. We believe that what really counts for localization is the scattering cross section and not the geometrical shape and real size of the scatterer. Therefore we will represent the dielectric particles located at the points $\vec{r}_a$ by *single* electric dipoles

$$\vec{\mathcal{P}}(\vec{r}) = \sum_a \vec{p}_a \, \delta(\vec{r} - \vec{r}_a), \tag{4}$$

with properly adjusted scattering properties. In this approximation strong localization of the scalar electromagnetic waves in two-dimensional random dielectric media has already been studied [22]. Since we are studying monochromatic fields only, then in our case the polarization of the medium is also the oscillatory function of time

$$\vec{P}(\vec{r}, t) = \mathrm{Re}\{\vec{\mathcal{P}}(\vec{r}) e^{-i\omega t}\}. \tag{5}$$

To investigate localization in the simple case of the dielectric media consisting of well separated particles it is sufficient to study the electromagnetic field in vacuum between those particles. It can be done very effectively using the Maxwell equations in the integral form [32]. Introducing the complex Hertz vector

$$\vec{\mathcal{Z}}(\vec{r}) = \int d^3r' \, \vec{\mathcal{P}}(\vec{r'}) \, \frac{e^{ik_0|\vec{r}-\vec{r'}|}}{|\vec{r}-\vec{r'}|}, \tag{6}$$

the field radiated by the finite dielectric medium (11) can be written in the form [33]

$$\vec{\mathcal{E}}^{(1)}(\vec{r}) = \vec{\nabla} \times [\vec{\nabla} \times \vec{\mathcal{Z}}(\vec{r})], \tag{7a}$$

$$\vec{\mathcal{H}}^{(1)}(\vec{r}) = -ik_0 \, \vec{\nabla} \times \vec{\mathcal{Z}}(\vec{r}). \tag{7b}$$

The total field may now be considered as the sum of the radiated field (7) and the free field $\vec{\mathcal{E}}^{(0)}(\vec{r}), \vec{\mathcal{H}}^{(0)}(\vec{r})$

$$\vec{\mathcal{E}}(\vec{r}) = \vec{\mathcal{E}}^{(0)}(\vec{r}) + \vec{\mathcal{E}}^{(1)}(\vec{r}), \tag{8a}$$

$$\vec{\mathcal{H}}(\vec{r}) = \vec{\mathcal{H}}^{(0)}(\vec{r}) + \vec{\mathcal{H}}^{(1)}(\vec{r}). \tag{8b}$$

Now we are ready to propose a definition of localized electromagnetic waves that resembles the definition of localized states in quantum mechanics and makes use of the analogy between the quantum-mechanical probability density and the energy density of the field. It seems natural to say that the monochromatic light wave is localized if the time-averaged energy density of the *total* field tends to zero far from a certain region of space

$$\mathcal{W}(\vec{r}) = \frac{1}{16\pi}\{|\vec{\mathcal{E}}(\vec{r})|^2 + |\vec{\mathcal{H}}(\vec{r})|^2\} \to 0 \quad \text{for} \quad |\vec{r}| \to \infty. \tag{9}$$

We used the fact that for rapidly oscillating monochromatic light waves (1) only time averages are measurable. Strictly speaking, to make the notion of the limit in Eq. (9) mathematically precise we must consider the energy density of the field outside the arbitrarily small but finite volumes surrounding the dipoles. In this way we avoid some infinities introduced by the point-scatterer approximation used. Let us emphasize that the condition (9) can be fulfilled *only* if the free field vectors $\vec{\mathcal{E}}^{(0)}(\vec{r})$ and $\vec{\mathcal{H}}^{(0)}(\vec{r})$ are zero everywhere. To see this let us observe that the localization condition (9) may be satisfied in systems (4) consisting of well separated dielectric particles only if the polarization of the medium vanishes at infinity, i.e., $\vec{\mathcal{P}}(\vec{r}) \to \vec{0}$ for $|\vec{r}| \to \infty$. Therefore, due to Eq. (6), the radiated field $\vec{\mathcal{E}}^{(1)}(\vec{r})$, $\vec{\mathcal{H}}^{(1)}(\vec{r})$ also tends to zero if $|\vec{r}| \to \infty$. Thus, if the light is localized according to the definition (9), the free field $\vec{\mathcal{E}}^{(0)}(\vec{r}), \vec{\mathcal{H}}^{(0)}(\vec{r})$ must also tend to zero in this limit. But it is known from the vector form of the Kirchhoff integral formula [34] that if the free field vanishes on a closed surface it is zero everywhere inside this surface.

It is interesting to note that in the absence of the free field the localization condition Eq. (9) is obviously fulfilled by the Hertz dipole [34] as well as other bounded radiating systems. However, we are interested in lossless media only, where localization is due to interference effects in elastic scattering of light by various dipoles. It is obvious that if the dielectric medium of the form of (4) is lossless, then the time-averaged field energy flux integrated over a surface surrounding arbitrary number of dipoles should vanish:

$$\int d\vec{s} \cdot \vec{\mathcal{S}}(\vec{r}) = \frac{c}{4\pi}\frac{1}{2}\mathrm{Re} \int d\vec{s} \cdot \{\vec{\mathcal{E}}(\vec{r}) \times \vec{\mathcal{H}}^*(\vec{r})\} = 0. \tag{10}$$

This condition immediately eliminates the above-mentioned radiating systems. Moreover, what is more important is that in Sec. III we will arrive at the conclusion that localized electromagnetic waves cannot exist in systems (4) consisting of a finite number of dipoles. In the case of bounded dielectric media the localization condition (9), which implies that free field must vanish everywhere, is compatible with the energy conservation condition (10) only if the total field is zero. However, as we will show in Sec. V, in bounded systems of randomly distributed dipoles there can still exist some special monochromatic waves for which the energy density of the field inside the medium is much larger than the energy of the incident wave in the stationary regime. We will call this phenomenon quasilocalization and prove that in the limit of the infinite number of dipoles it becomes practically indistinguishable from the perfect localization.

III. ELASTIC SCATTERING

Let us stress that in all experiments we can investigate only systems confined to certain finite regions of space. It is therefore reasonable to restrict our analysis to bounded media consisting of a finite number of dielectric particles. In this case the polarization (4) satisfies the condition

$$\vec{\mathcal{P}}(\vec{r}) = \vec{0} \quad \text{for} \quad |\vec{r}| > R. \tag{11}$$

Note that in the above formula we have explicitly introduced the characteristic length scale R, which will be used later on.

It turns out that in the case of bounded dielectric media (11) the energy conservation condition (10) leads to interesting restrictions imposed on the polarization $\vec{\mathcal{P}}(\vec{r})$. As we will see in the present section, the requirement that the time-averaged total-field energy flux integrated over a surface surrounding the considered bounded and lossless medium should vanish for the arbitrary incident wave $\vec{\mathcal{E}}^{(0)}(\vec{r})$, $\vec{\mathcal{H}}^{(0)}(\vec{r})$ determines uniquely the form of the dependence of the polarization of the medium on the field. The results of those general considerations dealing with elastic scattering by bounded dielectric media will be used in Sec. IV to derive the explicit form of the coupling between the pointlike three-dimensional dipole and the electric field of the light wave incident on it. Then we will be able to introduce discretized Maxwell equations that are one of the necessary ingredients of the definition of quasilocalized waves presented in Sec. V.

After inserting the formula (8), the expression (10) may be split into three terms. The first term

$$\int d\vec{s}\cdot \vec{\mathcal{S}}^{(1)}(\vec{r}) = \frac{c}{4\pi}\frac{1}{2}\mathrm{Re}\int d\vec{s}\cdot \{\vec{\mathcal{E}}^{(1)}(\vec{r}) \times \vec{\mathcal{H}}^{(1)*}(\vec{r})\} \tag{12}$$

corresponds to the time-averaged energy radiated by the medium per unit time. To obtain its explicit form, let us observe that far from the medium, i.e., for $|\vec{r}| \gg R$, the Hertz vector (6) can be approximated by

$$\vec{\mathcal{Z}}(\vec{r}) \approx \vec{\mathcal{P}}(k_0\,\vec{n})\,\frac{e^{ik_0|\vec{r}|}}{|\vec{r}|}, \tag{13}$$

where

$$\vec{\mathcal{P}}(\vec{k}) = \int d^3r\,\vec{\mathcal{P}}(\vec{r})\,e^{-i\,\vec{k}\cdot\vec{r}} \tag{14}$$

is the spatial Fourier transform of the polarization and

$$\vec{n} = \frac{\vec{r}}{|\vec{r}|} \tag{15}$$

is the unit vector pointing to the direction of observation. We see from Eq. (13) that for each direction of observation the far field radiated by the localized dielectric medium (11) looks like the field radiated by a certain Hertz dipole described by the polarization

$$\vec{\mathcal{P}}(\vec{r}) = \vec{\mathcal{P}}(k_0\,\vec{n})\,\delta(\vec{r}). \tag{16}$$

The Poynting vector of the field radiated by the Hertz dipole (16) in the far-field limit is given by (see, e.g., [32])

$$\vec{\mathcal{S}}^{(1)}(\vec{r}) = \vec{n}\,\frac{1}{8\pi}\frac{ck_0^4}{|\vec{r}|^2}|\vec{\mathcal{P}}_T(k_0\,\vec{n})|^2, \tag{17}$$

where

$$\vec{\mathcal{P}}_T(\vec{k}) = \vec{\mathcal{P}}(\vec{k}) - \frac{\vec{k}[\vec{k}\cdot\vec{\mathcal{P}}(\vec{k})]}{|\vec{k}|^2} \tag{18}$$

denotes the transverse part of the Fourier transform of the polarization. Integrating Eq. (17) over a sphere with radius $|\vec{r}|$ surrounding all sources we get the final expression for the average energy radiated by the dielectric medium (11)

$$\int d\vec{s}\cdot \vec{\mathcal{S}}^{(1)}(\vec{r}) = \frac{ck_0^4}{8\pi}\int d\Omega|\vec{\mathcal{P}}_T(k_0\,\vec{n})|^2. \tag{19}$$

Let us assume for a moment that in the bounded dielectric medium (11) there exists a localized [according to Eq. (9)] electromagnetic wave. As we have shown in Sec. II, the free field $\vec{\mathcal{E}}^{(0)}(\vec{r}), \vec{\mathcal{H}}^{(0)}(\vec{r})$ must vanish in this case. Therefore, to check the energy conservation condition (10) we have to integrate the time-averaged Poynting vector corresponding to the radiated field over a closed surface surrounding the medium:

$$\int d\vec{s}\cdot \vec{\mathcal{S}}^{(1)}(\vec{r}) = 0. \tag{20}$$

Inserting (19) into the condition (20), we see that the transverse part of the polarization of the medium vanishes on the light cone

$$\vec{\mathcal{P}}_T(\vec{k}) = \vec{0} \quad \text{for} \quad |\vec{k}| = k_0. \tag{21}$$

This condition cannot be fulfilled by any system (4) consisting of a finite number of dipoles. Indeed, the Fourier transform of the polarization is in this case given by a sum of finite number of plane waves

$$\vec{\mathcal{P}}(\vec{k}) = \sum_a \vec{p}_a\,e^{i\vec{r}_a\cdot\vec{k}} \tag{22}$$

and therefore the transverse part of it cannot vanish for all $|\vec{k}| = k_0$. So, as already mentioned at the end of Sec.

II, the localization condition (9) cannot be fulfilled in the case of spatially bounded lossless media consisting of a finite number of dielectric particles.

After this digression now we return again to the main theme of this section: elastic scattering. Let us take into account the presence of the nonzero free field and calculate the remaining terms in Eq. (10). The second term describes the total time-averaged energy flux integrated over a closed surface for the free field and thus vanishes

$$\frac{c}{4\pi}\frac{1}{2}\mathrm{Re}\int d\vec{s}\cdot\{\vec{\mathcal{E}}^{(0)}(\vec{r})\times\vec{\mathcal{H}}^{(0)*}(\vec{r})\}=0. \tag{23}$$

To calculate the last interference term

$$\frac{c}{4\pi}\frac{1}{2}\mathrm{Re}\int d\vec{s}\cdot\{\vec{\mathcal{E}}_0(\vec{r})\times\vec{\mathcal{H}}^{(1)*}(\vec{r})+\vec{\mathcal{E}}^{(1)}(\vec{r})\times\vec{\mathcal{H}}^{(0)*}(\vec{r})\}, \tag{24}$$

we use the identity (Lorentz theorem)

$$\vec{\nabla}\cdot\{\vec{\mathcal{E}}^{(0)}(\vec{r})\times\vec{\mathcal{H}}^{(1)*}(\vec{r})+\vec{\mathcal{E}}^{(1)*}(\vec{r})\times\vec{\mathcal{H}}^{(0)}(\vec{r})\}$$
$$=-ik_0\,4\pi\,\vec{\mathcal{P}}^*(\vec{r})\cdot\vec{\mathcal{E}}^{(0)}(\vec{r}), \tag{25}$$

which follows directly from the Maxwell equations. Integrating (25) over a volume containing the isolated part of the medium under consideration and calculating the real part we see that the Eq. (10) may be written in an equivalent form

$$\int d\vec{s}\cdot\vec{\mathcal{S}}^{(1)}(\vec{r})-\frac{1}{2}\,ck_0\,\mathrm{Re}\int d^3r\,\{i\vec{\mathcal{P}}^*(\vec{r})\cdot\vec{\mathcal{E}}^{(0)}(\vec{r})\}=0. \tag{26}$$

Thus, on average, the energy radiated by the medium must be equal to the energy given to the medium by the incident wave. The condition (26), together with Eq. (19), determines the relation between polarization and the electric field of the incident wave.

Note that the polarization of the dielectric medium

$$\vec{\mathcal{P}}(\vec{r})=\frac{\epsilon(\vec{r})-1}{4\pi}\vec{\mathcal{E}}(\vec{r}) \tag{27}$$

depends on the total field rather than on the field of the incident wave [34]. Also in quantum mechanics the potential operator acts on the full wave function. For the given incident wave one can obtain the total field solving the Maxwell equations. Therefore Eq. (27) may be written also in the form of Eq. (26). However, it is not necessary to identify the polarization vector $\vec{\mathcal{P}}$ with the *dielectric* polarization. Together with Maxwell equations in the integral form (7) it may be used as a convenient mathematical tool to study the radiation of electromagnetic waves by arbitrary sources [33]. Indeed the arbitrary current and charge densities obeying the continuity relation may be expressed in terms of the polarization vector

$$\vec{\jmath}=\partial_t\vec{P},\quad \rho=-\vec{\nabla}\cdot\vec{P}. \tag{28}$$

It is now clear that Eq. (26) describes elastic scattering of monochromatic light waves by arbitrary charges and currents. Indeed, there exist systems that can be described by Eq. (26) but do not consist of dielectric particles. As an example let us mention light scattering by perfectly conducting particles. The electric field of the light wave vanishes inside such particles but the polarization vector is not equal to zero, as opposed to Eq. (27).

IV. SYSTEM OF DIPOLES

It is known that several mathematical problems emerge in the formulation of interactions of pointlike dielectric particles with electromagnetic waves [19–21]. Instead of applying several complicated regularization procedures, we will show that it is possible to analyze light scattering by pointlike dielectric particles as a special case of general considerations dealing with elastic scattering of electromagnetic waves by arbitrary charges and currents presented in Sec. III. This treatment may be viewed as a generalization of the results obtained in our previous paper [22] to the case of the vector field. Previous results corresponding to the two-dimensional case were based on the Kirchhoff integral formula for scalar waves. We are interested in lossless media only, where localization is due to interference effects in elastic scattering of light by various dipoles. Therefore the time-averaged Poynting vector integrated over a surface surrounding each dipole must vanish for the arbitrary wave incident on this dipole. As shown in Sec. III, this condition will be fulfilled if the dipole moments $\vec{p}_a$ depend on the electric field of the wave incident on them.

To obtain an explicit form of the coupling let us recall the formula for the energy radiated on average by the Hertz dipole

$$\int d\vec{s}\cdot\vec{\mathcal{S}}^{(1)}(\vec{r})=\frac{1}{3}ck_0^4|\vec{p}|^2, \tag{29}$$

which can be found, for example, in [32]. Inserting the above expression and Eq. (4), we may rewrite the condition (26) in the form

$$\left|ik_0^3\,\vec{p}_a+\frac{3}{4}\,\vec{\mathcal{E}}'(\vec{r}_a)\right|^2=\left|\frac{3}{4}\,\vec{\mathcal{E}}'(\vec{r}_a)\right|^2, \tag{30}$$

where the field acting on the ath dipole

$$\vec{\mathcal{E}}'(\vec{r}_a)=\vec{\mathcal{E}}^{(0)}(\vec{r}_a)+\sum_{b\neq a}\vec{\mathcal{E}}_b(\vec{r}_a) \tag{31}$$

is the sum of the free field and waves radiated by all other dipoles

$$\vec{\mathcal{E}}_a(\vec{r})=\frac{2}{3}\,ik_0^3\,\tilde{g}(\vec{r}-\vec{r}_a)\cdot\vec{p}_a, \tag{32}$$

which are expressed by the Green tensor (see, e.g., [32])

$$\tilde{g}(\vec{r}) = -i\frac{3}{2}\frac{e^{ik_0|\vec{r}|}}{k_0|\vec{r}|}\left\{\left(\frac{3}{(k_0|\vec{r}|)^2} - i\frac{3}{k_0|\vec{r}|} - 1\right)\frac{\vec{r}\,\vec{r}}{|\vec{r}|^2} - \left(\frac{1}{(k_0|\vec{r}|)^2} - i\frac{1}{k_0|\vec{r}|} - 1\right)\right\}. \tag{33}$$

Assuming that the vector on the left-hand side of Eq. (30) is a function of the vector on the right-hand side and that the dielectric particles modeled by the dipoles are spherically symmetrical we get

$$i\pi\, k_0^2\, \vec{p}_a = \frac{1}{2}(e^{i\phi_a} - 1)\vec{\mathcal{E}}'(\vec{r}_a). \tag{34}$$

Thus, to ensure conservation of energy, the dipole moments are coupled to the electric field of the incident wave by complex "polarizability" $(e^{i\phi_a} - 1)/2$, which can take values from a circle on the complex plane.

Note that the case $\phi_a = \pi$ corresponds to the internal resonance of the dielectric particle described by a dipole. The total scattering cross section takes then its maximal value. Indeed, dividing Eq. (26) by the intensity of a plane wave given by [32]

$$I = \frac{c}{4\pi}|\vec{\mathcal{E}}^{(0)}(\vec{r}_a)|^2 \tag{35}$$

and inserting Eq. (34) we obtain the explicit formula for the total scattering cross section σ_a of the ath dipole

$$k_0^2\,\sigma_a = 3\pi\frac{1 - \cos\phi_a}{2}. \tag{36}$$

Inserting Eq. (32) into (31), using (34), and introducing the convenient notation

$$G_{ab} = \begin{cases} \tilde{g}(\vec{r}_a - \vec{r}_b) & \text{for } a \neq b \\ 0 & \text{for } a = b, \end{cases} \tag{37}$$

we obtain finally the system of equations

$$\vec{\mathcal{E}}'(\vec{r}_a) = \vec{\mathcal{E}}^{(0)}(\vec{r}_a) + \frac{1}{2}\sum_b G_{ab}(e^{i\phi_b} - 1)\vec{\mathcal{E}}'(\vec{r}_b) \tag{38}$$

determining the field acting on each dipole $\vec{\mathcal{E}}'(\vec{r}_a)$ for a given free field $\vec{\mathcal{E}}^{(0)}(\vec{r}_a)$. If we solve it and calculate the dipole moments according to Eq. (34) we are able to find the electromagnetic field everywhere in space using the Maxwell equations in the integral form. It follows from the considerations presented in Sec. II that if the system of dipoles (4) ensures localization of the light wave, then the system of equations (38) has a nonzero solution for vanishing free field $\vec{\mathcal{E}}^{(0)}(\vec{r}) = \vec{0}$. However, as shown in Sec. III, this may be possible only in the case of an infinite number of dipoles.

V. QUASILOCALIZATION

As follows from previous considerations, the perfectly localized states do not exist in systems (4) consisting of a finite number of three-dimensional dipoles. Nevertheless, in the case of bounded media there can exist some monochromatic resonance waves for which the field energy density inside the medium is much larger than the energy density of the incident wave in the stationary regime. We will call this phenomenon quasilocalization in contrast to the prefect localization that can take place only in infinite random media. Note that in the experiment on microwave localization in two-dimensional structures [18] such quasilocalized modes have been identified be direct measurement of the squared electric field. In contrast to this simple case, in three-dimensional media the existence of such quasilocalized modes can be proven only by observing resonance effects in transmission. In this section we present the detailed analysis of the quasilocalized waves existing in systems of a finite number of randomly distributed three-dimensional dipoles.

Let us study eigenmodes of the system of equations (38) determining field acting on each dipole

$$\lambda_j\, \vec{\mathcal{E}}'_j(\vec{r}_a) = \vec{\mathcal{E}}^{(0)}_j(\vec{r}_a). \tag{39}$$

Obviously eigenvalues λ_j corresponding to eigenvectors describing perfectly localized light waves should be equal to zero. But it is impossible in systems (4) consisting of a finite number of dipoles. However, in this case the modulus of some eigenvalues can be very small. This means that the field incident on each dipole $|\mathcal{E}'_j(\vec{r}_a)|$ is large compared to the free field $|\mathcal{E}^{(0)}_j(\vec{r}_a)|^2$ calculated at the dipole. Thus the time-averaged energy density in the medium under consideration can indeed be much greater than the energy density in the surrounding free space. It is therefore natural to call a light wave corresponding to such an eigenvector a quasilocalized wave. Of course the free field is not completely determined by specifying only its values at the dipoles according to Eq. (39). However, we believe that it may be constructed in such a way that the time-averaged energy density of the free field will not exhibit local minima at the dipoles, so that the energy density outside the medium can still be much smaller than the localized energy density inside.

The crucial point now is how each dipole should be coupled to the electromagnetic field. It turns out that, assuming that all dipoles are identical

$$\phi_a = \phi, \tag{40}$$

for a given distribution of dipoles we can easily choose their scattering properties ϕ to minimize the absolute value of a certain eigenvalue $|\lambda_j|$. Indeed, in the case of the same scattering properties of all dipoles (40) the eigenvalues

$$\lambda_j(\phi) = 1 - \frac{e^{i\phi} - 1}{2}\lambda'_j \tag{41}$$

are expressed by eigenvalues λ'_j of the G matrix (37) that depend only on positions of the dipoles. Note that in this case eigenvectors of the system (38) are simultaneously eigenvectors of the G matrix.

As a simple example let us now consider a system of $N = 100$ dipoles (4) distributed randomly in a sphere with uniform density $n = 1$ per wavelength cubed. We have calculated and diagonalized numerically the G matrix (37) of rank $3 \times N$ describing this situation and checked if a certain eigenvalue of the system of equations (38) can approach zero. To do this, for each phase ϕ we have calculated the absolute value of each eigenvalue λ_j according to Eq. (41) and chosen the smallest one from all eigenmodes

$$\Lambda(\phi) = \min_j |\lambda_j(\phi)|. \tag{42}$$

The function $\Lambda(\phi)$ is plotted in Fig. 1. In close analogy with our previous considerations, perfect localization is impossible in systems (4) consisting of a finite number of three-dimensional dipoles. However, the modulus of an eigenvalue can be very small; e.g., in the local minimum near $\phi \simeq -0.27$, we have obtained $\Lambda \propto 10^{-2}$.

Let us emphasize that, as illustrated in Fig. 1, monochromatic resonance waves with energy density well localized inside the medium occur when considering one sample and then varying ϕ. We would now like to fix first the parameter ϕ and then look at quasilocalized waves in a corresponding typical medium. To do this we have diagonalized numerically the G matrix (37) for 10^4 different random distributions of the dipoles. For each distribution we have calculated the minimal absolute value of an eigenvalue (42) as a function of the phase ϕ. The contour plot of the corresponding two-dimensional probability distribution is presented in Fig. 2. This plot can be considered as the solid line from Fig. 1 spread out over different samples. We see that although the positions of the minima in Fig. 1 corresponding to resonances are strongly dependent on different distributions of the dipoles, these minima still appear in the same region of ϕ. This means that after choosing ϕ from this region, for almost any realization of the random dielectric medium there exists a quasilocalized wave. It is also interesting to stress that, according to Fig. 2, the phase ϕ corresponding to optimal localization is close to zero. Thus the scattering cross sections of the dipoles (36) do not take their maximal values. It means that localization of light in a system of dielectric particles occurs for different values of ϕ than the internal resonances of those particles.

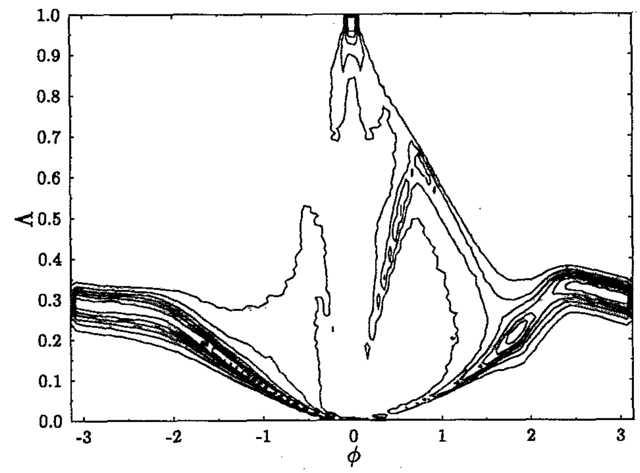

FIG. 2. Contour plot of the probability distribution of solid curves from Fig. 1 calculated for 10^4 systems of $N = 100$ dipoles distributed randomly in a sphere with the density $n = 1$ dipoles per wavelength cubed.

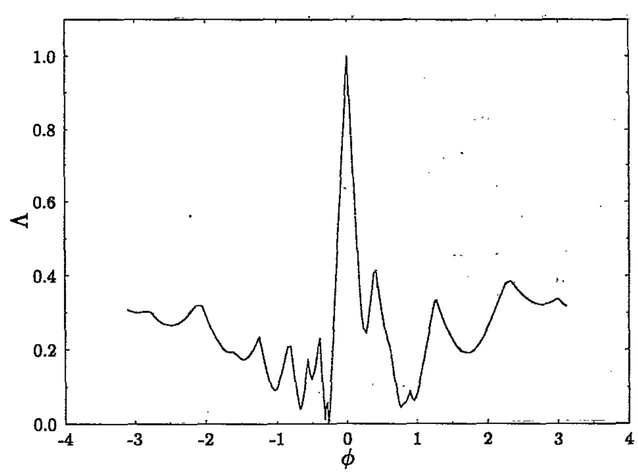

FIG. 1. Minimal modulus of the eigenvalue Λ plotted as a function of the phase ϕ that describes scattering properties of the single system of $N = 100$ dipoles. Dipoles are distributed randomly in a sphere with the density $n = 1$ dipoles per wavelength cubed.

It follows from inspection of Fig. 1 that if $\vec{r}_a$ are given, then there exists a certain phase $\phi^{(0)}$ describing the scattering properties of the dipoles for which the modulus of a certain eigenvalue $\lambda_{(0)}$ takes a minimal possible value $\Lambda^{(0)}$. To find the position of this sharp resonance let us make the following observation. It follows from Eq. (41) that for each eigenvector of the system of equations (38) there exists a certain phase ϕ describing the scattering properties of the dipoles

$$\phi^{(j)} = \arg\left(1 + \frac{\lambda'_j}{2}\right) - \arg\left(\frac{\lambda'_j}{2}\right), \tag{43}$$

for which the modulus of the corresponding eigenvalue λ_j takes a minimal value $\Lambda^{(j)}$ given by

$$\Lambda^{(j)} = \left| \left|1 + \frac{\lambda'_j}{2}\right| - \left|\frac{\lambda'_j}{2}\right| \right| = |\lambda_j(\phi^{(j)})| \leq |\lambda_j(\phi)|. \tag{44}$$

According to Eq. (44) we may now easily calculate the minimal values of modulus for all eigenvalues and choose the smallest one

$$\Lambda^{(0)} = \min_j \Lambda^{(j)}. \tag{45}$$

We have repeated this procedure for several G matrices (37) describing different systems of dipoles (4) distributed randomly in a sphere with uniform density and calculated the average value of $\Lambda^{(0)}$. The results are plotted in Fig. 3 as a function of the number of dipoles N for various numbers of dipoles n per wavelength cubed. We see again that perfect localization is impossible in systems (4) of a finite number of dipoles. However, for a fixed density of the medium under consideration quasilocalization becomes better ($\Lambda^{(0)}$ decreases) for the increas-

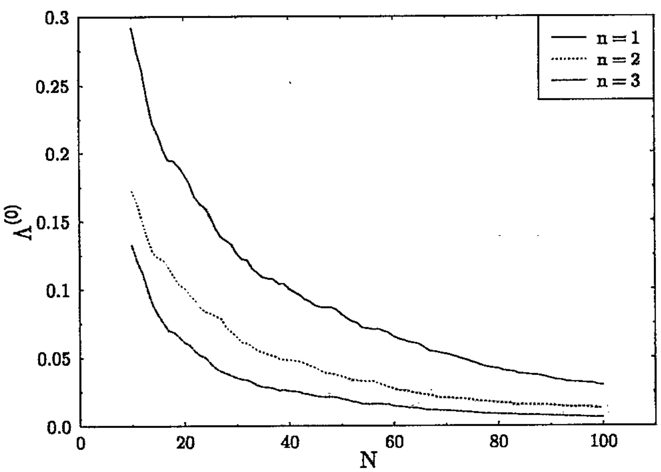

FIG. 3. Minimal possible modulus of the eigenvalue $\Lambda^{(0)}$ as a function of the total number of dipoles N for various numbers n of dipoles per wavelength cubed. This plot has been averaged over 10^3 random distributions of dipoles placed in a sphere with uniform density.

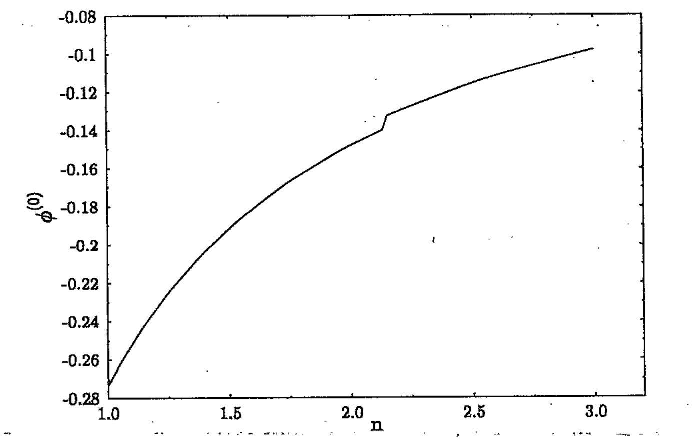

FIG. 5. Minimal possible modulus of the eigenvalue $\Lambda^{(0)}$ corresponding to the phase $\phi^{(0)}$ plotted in Fig. 4.

ing number of dipoles N.

In fact, for a fixed system of dipoles given by Eq. (34), Eq. (45), together with Eq. (43) determining the corresponding phase $\phi^{(0)}$, defines the dependence of the scattering properties of dipoles on the frequency of localized field. In Fig. 4 we have plotted the phase $\phi^{(0)}$ ensuring the best possible localization in a system of $N = 100$ dipoles distributed randomly in a sphere as a function of the number of dipoles n per wavelength cubed. Therefore we have constructed a model of a system of dielectric particles with the frequency-dependent permeability $\epsilon(\omega)$ chosen to ensure the best possible localization for all frequencies from some region for which $\epsilon(\omega)$ can be real. Indeed, as shown in Fig. 5, in such an abstract random medium quasilocalization takes place practically for all sufficiently large densities (i.e., $n > 1$). Of course, in any realistic experiment the dielectric particles must have a finite diameter to provide a scattering cross section sufficiently large for localization. It imposes the practical limit on the maximum density.

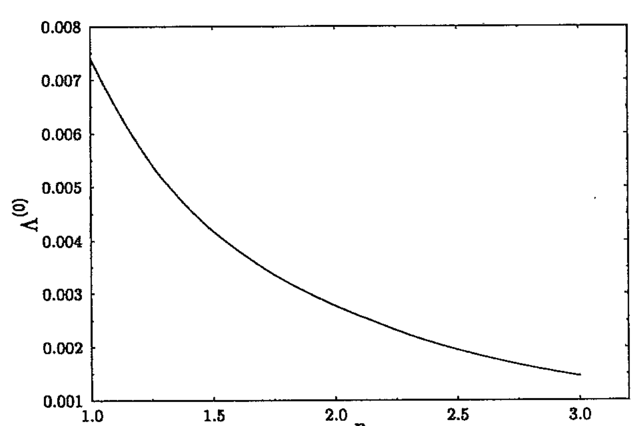

FIG. 4. Phase $\phi^{(0)}$ for the single system of $N = 100$ dipoles distributed randomly in a sphere as a function of the number of dipoles n per wavelength cubed.

VI. SUMMARY

We developed a simple theory of elastic scattering of electromagnetic waves by dielectric particles that is based on the optical theorem. In our treatment the internal resonances of the dielectric particles are incorporated in a unified way. This treatment may be viewed as a generalization of the results obtained in our previous paper [22] (which were based on the Kirchhoff integral formula for scalar waves) to the case of the vector fields. The presented approach to localization of electromagnetic waves in three dimensions is based on Maxwell equations in integral form. The dielectric medium providing localization is modeled by a system of discrete dipoles. Each dipole, with properly adjusted scattering properties, replaces a single dielectric particle. In this treatment the scattering properties of the medium may be eliminated from the considerations and the best possible localization for a given distribution of particles can be studied. It was shown that, in media consisting of a finite number of dielectric particles, localization of light is impossible. However, for almost any random distribution of the dielectric particles there can exist certain monochromatic resonance waves with energy density well localized inside the medium. If the total number of scattering particles tends to infinity, those resonances become localized states. Similar effects have been experimentally studied by direct measurement of the energy density of the field in a two-dimensional random collection of dielectric cylinders [18].

It was shown that in our abstract dielectric medium with specially chosen frequency-dependent permeability, localization of light is easier to achieve for large numbers of dipoles per wavelength cubed. However, in any actual experiment the dielectric particles must have a sufficiently large diameter to provide a scattering cross section suitable for localization. This requirement imposes a practical limit on the maximal density. On the other hand, in the limit of small wavelengths, the point-scatterer approximation used becomes invalid. Therefore our results agree completely with the common belief (see, e.g., [35,29]) that in three-dimensional media light localization is possible only in a certain frequency window.

Our considerations suggest that localization of light in three-dimensional systems consisting of randomly distributed dielectric particles occurs for ranges of parameters different from the internal resonances of those particles. They also provide additional insight into the problem of localization on resonating scatterers [36,37]. It seems that resonating scatterers do not provide localization. It is in agreement with results based on the analysis of the dependent-scattering effects in the amplitude Green's functions [19,20], where the essential argument was the overlap of optical volumes of the scatterers.

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