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Bands of Localized Electromagnetic Waves in 3D Random Media

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Anderson localization of electromagnetic waves in three-dimensional disordered dielectric structures is studied using a simple yet realistic theoretical model. An effective approach based on analysis of probability distributions, not averages, is developed. The disordered dielectric medium is modeled by a system of randomly distributed electric dipoles. Spectra of certain random matrices are investigated and the possibility of appearance of the continuous band of localized waves emerging in the limit of an infinite medium is indicated. It is shown that localization could be achieved without tuning the frequency of monochromatic electromagnetic waves to match the internal (Mie-type) resonances of individual scatterers. A possible explanation for the lack of experimental evidence for strong localization in 3D as well as suggestions how to make localization experimentally feasible are also given. Rather peculiar requirements for setting in localization in 3D as compared to 2D are indicated.

1. INTRODUCTION

The last conference directed by the late Asim Barut was the NATO ASI *Quantum Electrodynamics and Electron Theory: 100 Years Later*, held in Edirne, Turkey, in 1994. One of the authors of the present paper (A.O.) had an opportunity to be there and to present a lecture on localization of light. In the meantime some of our results concerning 2D media[1–3] as well as some preliminary 3D calculations[4] were published. It is our pleasure to admit that their final form benefited much from the warm and constructive feedback we received during that meeting. Still there are many open problems and interesting questions that remain unanswered. In this paper we go ahead with 3D disordered dielectric structures presenting both new results of numerical simulations and their physical interpretation. Disordered dielectric

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media are a subject of intensive experimental and theoretical studies. Electromagnetic waves propagating in these structures mimic, to a reasonable extent, the behavior of electrons in disordered semiconductors. Many ideas concerning transport properties of light and microwaves in such media exploit the well-developed theoretical methods and concepts of solid-state physics. Let us mention, e.g., the concept of electron localization in noncrystalline systems such as amorphous semiconductors or disordered insulators. According to Anderson,[5] an entire *band* of electronic states can be spatially localized in a sufficiently disordered infinite material. Before this discovery, it was believed that electronic states in infinite media are either extended, by analogy with the Bloch picture for crystalline solids, or are localized around *isolated* spatial regions such as surfaces and impurities.[6] A very common theoretical approach in investigations of Anderson localization in solid-state physics is to study the transport equation for the ensemble-averaged squared modulus of the wave function.[7-9] Under some assumptions such a transport equation can be converted into a diffusion equation. The diffusion constant D becomes a parameter monitoring behavior of the system. Strong localization is achieved when the diffusion constant in the scattering medium becomes zero. When the fluctuations of the electronic static potential become large enough, the wavefunction ceases to diffuse and becomes localized. Thus the Anderson transition may be viewed as a transition from particlelike behavior described by the diffusion equation to wavelike behavior described by the Schrödinger equation which results in localization.[10]

As interference is the common property of all wave phenomena, the quest for some analogs of electron localization for other types of waves has been undertaken and many generalizations of electron localization exist, especially in the realm of electromagnetic waves.[11-14] So-called weak localization of electromagnetic waves manifesting itself as enhanced coherent backscattering is presently relatively well understood theoretically[15-17] and established experimentally.[18-20] The question is whether interference effects in 3D random dielectric media can reduce the diffusion constant to zero leading to strong localization. The crucial parameter is the mean free path l which should be rather short.[21-23] Once $l < l_{cr}$, all states of the system will be localized. The Ioffe–Regel criterion gives $kl_{cr} = 1$ and, indeed, the existence of delocalized states at $kl < 1$ seems very unlikely. It is possible, however, that the localization transition might occur earlier. It seems that a suspension of TiO_2 spheres in air is the system in which the shortest l-values for visible light may be realized in practice. However, even in the samples with strongest randomness, l is still higher by a factor of 10 than the Ioffe–Regel value for l_{cr}.[24] Despite the observation of scale dependence of the diffusion constant in such media, which may be considered as a reasonable indication of Anderson

transition, there still is no convincing experimental demonstration that strong localization could be possible in 3D random dielectric structures. Such a demonstration has only been given for two dimensions, where strong localization takes place for arbitrarily small values of the mean free path (if the medium is sufficiently large). The strongly scattering medium has been provided by a set of dielectric cylinders randomly placed between two parallel aluminum plates on half the sites of a square lattice.[25]

Better understanding of this interesting effect requires sound theoretical models. Such models should be based directly on the Maxwell equations and they should be simple enough to provide calculations without too many approximations. There is a temptation to immediately apply averaging procedures as soon as "disorder" is introduced into the model. Averaging of the scattered intensity over some random variable leads to a transport theory.[26] But "there is a very important and fundamental truth about random systems we must always keep in mind: no real atom is an average atom, nor is an experiment done on an ensemble of samples."[27] We always deal with a specific example of the disordered system. Therefore what we really need to properly understand the existing experimental results are probability distributions, not averages.

In this paper we develop a theoretical model of the Anderson localization of electromagnetic waves in 3D dielectric media without using any averaging procedures. We restrict ourselves to the study of the properties of the stationary solutions of the Maxwell equations in random suspensions of dielectric spheres. By investigating the band of localized waves in such media, we discuss the possible origins of those experimental difficulties and shed new light on the problem of resonant scattering. We suggest that the null results of 3D experiments are *not* necessarily due to the fact that we are not able to satisfy the Ioffe–Regel criterion. On the contrary, we show that trying to satisfy this criterion by enhancement of the scattering cross section of individual scatterers via tuning the frequency to the internal (Mie-like) resonances, we may leave the region of parameters that is optimal for localization in 3D media.

2. LIPPMANN–SCHWINGER EQUATIONS FOR ELECTROMAGNETIC WAVES

The standard approach to localization of electromagnetic waves[10, 22] is based on the similarities between the time-independent Schrödinger equation

$$\left\{-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right\}\psi(\mathbf{r}) = E\psi(\mathbf{r}) \tag{1}$$

and the Helmholtz equation for the monochromatic electric field amplitude in an isotropic lossless dielectric

$$\nabla\times\nabla\times\mathscr{E}(\mathbf{r})+k^2[1-\varepsilon(\mathbf{r})]\mathscr{E}(\mathbf{r})=k^2\mathscr{E}(\mathbf{r}) \tag{2}$$

The term $k^2[1-\varepsilon(\mathbf{r})]$ corresponds to the potential $V(\mathbf{r})$ providing localization of the electron wavefunction, and the squared wave number in vacuum $k^2=\omega^2/c^2$ plays a role analogous to the energy eigenvalue E.

Of course, apart from remarkable similarities, there are also striking differences between quantum particles and electromagnetic waves.[28, 29] Very different is, e.g., the long-wavelength limit of elastic scattering. For electrons we have mainly s-wave scattering which is spatially isotropic and wavelength independent. For electromagnetic waves we observe p-wave scattering. In this case there is forward–backward symmetry but scattering is nonisotropic. In addition, in the long-wavelength limit, the cross-section for scattering of electromagnetic waves shows the well-known λ^{-4} dependence. In inelastic scattering electrons change their energy but their total number is conserved. For electromagnetic waves we have strong absorption and the intensity decreases. Moreover electrons are described by scalar wave functions (or two-component spinors if the spin is included). To describe correctly localization of electromagnetic waves we need to consider, in general, three-dimensional vector fields.

Now we are ready to present a possible definition of localized electromagnetic waves which resembles the definition of localized states in quantum mechanics and makes use of the analogy between the quantum-mechanical probability density and the energy density of the field. In general, the electric field $\mathbf{E}(\mathbf{r}, t)$ cannot be interpreted as the probability amplitude.[29] The correct equivalent of the quantum-mechanical probability density is rather the energy density of the field and not the squared electric field. Therefore it seems natural to say that the monochromatic electromagnetic wave

$$\mathbf{E}(\mathbf{r}, t)=\mathrm{Re}\{\mathscr{E}(\mathbf{r})\,e^{-i\omega t}\} \tag{3a}$$

$$\mathbf{H}(\mathbf{r}, t)=\mathrm{Re}\{\mathscr{H}(\mathbf{r})\,e^{-i\omega t}\} \tag{3b}$$

is localized if the time-averaged energy density of the total field tends to zero far from a certain region of space

$$\mathscr{W}(\mathbf{r})=\frac{1}{16\pi}\{|\varepsilon(\mathbf{r})\,\mathscr{E}(\mathbf{r})|^2+|\mathscr{H}(\mathbf{r})|^2\}\to 0, \qquad \text{for} \quad |\mathbf{r}|\to\infty \tag{4}$$

We used the fact that for rapidly oscillating monochromatic electromagnetic waves (4) only time averages are measurable.

The total field

$$\mathscr{E}(\mathbf{r}) = \mathscr{E}^{(0)}(\mathbf{r}) + \mathscr{E}^{(1)}(\mathbf{r}) \tag{5a}$$

$$\mathscr{H}(\mathbf{r}) = \mathscr{H}^{(0)}(\mathbf{r}) + \mathscr{H}^{(1)}(\mathbf{r}) \tag{5b}$$

may be considered as the sum of some incident free field $\mathscr{E}^{(0)}(\mathbf{r})$, $\mathscr{H}^{(0)}(\mathbf{r})$, which obeys the Maxwell equations in vacuum, and waves scattered by the various parts of the medium[30]:

$$\mathscr{E}^{(1)}(\mathbf{r}) = \nabla\times\nabla\times\mathscr{Z}(\mathbf{r}) - 4\pi\mathscr{P}(\mathbf{r}) \tag{6a}$$

$$\mathscr{H}^{(1)}(\mathbf{r}) = -ik\nabla\times\mathscr{Z}(\mathbf{r}) \tag{6b}$$

The electric Hertz potential

$$\mathscr{Z}(\mathbf{r}) = \int d^3r'\,\mathscr{P}(\mathbf{r}')\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} \tag{7}$$

is expressed by the polarization of the dielectric medium

$$\mathscr{P}(\mathbf{r}) = \frac{\varepsilon(\mathbf{r})-1}{4\pi}\mathscr{E}(\mathbf{r}) \tag{8}$$

The system of equations (5)–(8) determines the electromagnetic field $\mathscr{E}(\mathbf{r})$, $\mathscr{H}(\mathbf{r})$ everywhere in space for a given field $\mathscr{E}^{(0)}(\mathbf{r})$, $\mathscr{H}^{(0)}(\mathbf{r})$ of the free wave incident on the system. Analogous relationships between the stationary outgoing wave and the stationary incoming wave are known in the general scattering theory as the Lippmann–Schwinger equations.[31] A way of dealing with bound states in the formalism of the Lippman–Schwinger equation is to solve it as a homogeneous equation, i.e., for the incoming wave (in our case $\mathscr{E}^{(0)}(\mathbf{r})$) equal to zero.

3. POINT-SCATTERER APPROXIMATION

Usually localization of light is studied experimentally in microstructures consisting of dielectric spheres with diameters and mutual distances that are comparable to the wavelength.[23] It is well known that the theory of multiple scattering of light by dielectric particles is tremendously simplified in the limit of point scatterers. In principle, this approximation is justified only when the size of the scattering particles is much smaller than the wavelength. In practical calculations, however, many multiple-scattering effects can be obtained qualitatively for coupled electrical dipoles.

Examples are universal conductance fluctuations,[34] enhanced backscattering,[35] and dependent scattering.[36] We believe that what really counts for localization is the scattering cross-section and not the geometrical shape and real size of the scatterer. Therefore we will represent the dielectric particles located at the points $\mathbf{r}_a$ by *single* electric dipoles

$$\mathscr{P}(\mathbf{r}) = \sum_a \mathbf{p}_a \delta(\mathbf{r} - \mathbf{r}_a) \tag{9}$$

with properly adjusted scattering properties.

It is known that several mathematical problems emerge in the formulation of interactions of pointlike dielectric particles with electromagnetic waves.[36-38] Instead of applying several complicated regularization procedures we will show that it is possible to analyze light scattering by pointlike dielectric particles as a special case of general considerations dealing with elastic scattering of electromagnetic waves. To use safely the point dipole approximation it is essential to use a representation for the scatterers that fulfills the optical theorem rigorously and conserves energy in the scattering processes. Therefore, the time-averaged field energy flux integrated over a surface surrounding an arbitrary part of the medium should vanish:

$$\int d\mathbf{s} \cdot \mathscr{S}(\mathbf{r}) = \frac{c}{4\pi} \frac{1}{2} \operatorname{Re} \int d\mathbf{s} \cdot \{\mathscr{E}(\mathbf{r}) \times \mathscr{H}^*(\mathbf{r})\} = 0 \tag{10}$$

After inserting the formula (7), the expression (13) may be split into three terms. The first term

$$\int d\mathbf{s} \cdot \mathscr{S}^{(1)}(\mathbf{r}) = \frac{c}{4\pi} \frac{1}{2} \operatorname{Re} \int d\mathbf{s} \cdot \{\mathscr{E}^{(1)}(\mathbf{r}) \times \mathscr{H}^{(1)*}(\mathbf{r})\} \tag{11}$$

corresponds to the time-averaged energy radiated by the medium per unit time. The second term describes the total time-averaged energy flux integrated over a closed surface for the free field and thus vanishes:

$$\frac{c}{4\pi} \frac{1}{2} \operatorname{Re} \int d\mathbf{s} \cdot \{\mathscr{E}^{(0)}(\mathbf{r}) \times \mathscr{H}^{(0)*}(\mathbf{r})\} = 0 \tag{12}$$

To calculate the last interference term

$$\frac{c}{4\pi} \frac{1}{2} \operatorname{Re} \int d\mathbf{s} \cdot \{\mathscr{E}^{(0)}(\mathbf{r}) \times \mathscr{H}^{(1)*}(\mathbf{r}) + \mathscr{E}^{(1)}(\mathbf{r}) \times \mathscr{H}^{(0)*}(\mathbf{r})\} \tag{13}$$

we use the following identity (Lorentz theorem):

$$\nabla \cdot \{\mathscr{E}^{(0)}(\mathbf{r}) \times \mathscr{H}^{(1)*}(\mathbf{r}) + \mathscr{E}^{(1)*}(\mathbf{r}) \times \mathscr{H}^{(0)}(\mathbf{r})\} = -ik\, 4\pi \mathscr{P}^{*}(\mathbf{r}) \cdot \mathscr{E}^{(0)}(\mathbf{r}) \quad (14)$$

which follows directly from the Maxwell equations

$$\begin{cases} \nabla \times \mathscr{E}^{(0)}(\mathbf{r}) = ik\, \mathscr{H}^{(0)}(\mathbf{r}) \\ \nabla \times \mathscr{H}^{(0)}(\mathbf{r}) = -ik\, \mathscr{E}^{(0)}(\mathbf{r}) \end{cases} \qquad \begin{cases} \nabla \times \mathscr{E}^{(1)}(\mathbf{r}) = ik\, \mathscr{H}^{(1)}(\mathbf{r}) \\ \nabla \times \mathscr{H}^{(1)}(\mathbf{r}) = -ik\, \mathscr{E}^{(1)}(\mathbf{r}) + ik\, 4\pi \mathscr{P}(\mathbf{r}) \end{cases} \quad (15)$$

Integrating (14) over a volume containing the part of the medium under consideration and calculating the real part we see that Eq. (10) may be written in an equivalent form:

$$\int d\mathbf{s} \cdot \mathscr{S}^{(1)}(\mathbf{r}) - \tfrac{1}{2} ck\, \mathrm{Re} \int d^3 r\, \{i \mathscr{P}^{*}(\mathbf{r}) \cdot \mathscr{E}^{(0)}(\mathbf{r})\} = 0 \quad (16)$$

Thus on average the energy radiated by the medium must be equal to the energy given to the medium by the incident wave.

Now, let us insert Eq. (9) into the energy conservation condition (16) and perform the integration over a surface surrounding the ath dipole only. Recalling the formula for the energy radiated on average by the Hertz dipole[30]:

$$\int d\mathbf{s} \cdot \mathscr{S}^{(1)}(\mathbf{r}) = \tfrac{1}{3} ck^4 |\mathbf{p}_a|^2 \quad (17)$$

we arrive at

$$|ik^3 \mathbf{p}_a + \tfrac{3}{4} \mathscr{E}'(\mathbf{r}_a)|^2 = |\tfrac{3}{4} \mathscr{E}'(\mathbf{r}_a)|^2 \quad (18)$$

where the field acting on the ath dipole

$$\mathscr{E}'(\mathbf{r}_a) = \mathscr{E}^{(0)}(\mathbf{r}_a) + \sum_{b \neq a} \mathscr{E}_b(\mathbf{r}_a) \quad (19)$$

is the sum of the free field and waves scattered by all other dipoles (which is a special case of Eq. (6)):

$$\mathscr{E}_a(\mathbf{r}) = \nabla \times \nabla \times \mathbf{p}_a \frac{e^{ik|\mathbf{r} - \mathbf{r}_a|}}{|\mathbf{r} - \mathbf{r}_a|} \quad (20)$$

Assuming that the vector on the left-hand side of Eq. (18) is a function of the vector on the right-hand side and that the dielectric particles modeled by the dipoles are spherically symmetrical, we get

$$\frac{2}{3} ik^3 \mathbf{p}_a = \frac{e^{i\phi} - 1}{2} \mathscr{E}'(\mathbf{r}_a) \tag{21}$$

Thus, to provide conservation of energy, the dipole moments must be $\mathbf{p}_a$ coupled to the electric field of the incident wave $\mathscr{E}'(\mathbf{r}_a)$ by complex "polarizability" $(e^{i\phi} - 1)/2$, which can take values from a circle on the complex plane.

To get some insight into the physical meaning of the parameter ϕ from Eq. (21) let us observe that it is directly related to the total scattering cross-section σ of an individual dielectric sphere represented by the single dipole. Indeed, dividing Eq. (16) by the intensity of a plane wave given by[30]

$$I = \frac{c}{4\pi} |\mathscr{E}^{(0)}(\mathbf{r}_a)|^2 \tag{22}$$

and inserting Eq. (24) we obtain the explicit formula for the scattering cross-section:

$$k^2 \sigma = \frac{3\pi}{2} (1 - \cos \phi) \tag{23}$$

Now, inserting Eq. (20) into (19) and using (21) it is easy to obtain finally the system of linear equations:

$$\sum_b \vec{M}_{ab} \cdot \mathscr{E}'(\mathbf{r}_b) = \mathscr{E}^{(0)}(\mathbf{r}_a) \tag{24}$$

determining the field acting on each dipole $\mathscr{E}'(\mathbf{r}_a)$ for a given free field $\mathscr{E}^{(0)}(\mathbf{r}_a)$. If we solve it and use again Eqs. (21) and (20) then we are able to find the electromagnetic field everywhere in space (but outside of the dipoles):

$$\mathscr{E}(\mathbf{r}) = \mathscr{E}^{(0)}(\mathbf{r}) + \mathscr{E}_a(\mathbf{r}), \qquad \text{for} \quad \mathbf{r} \neq \mathbf{r}_a \tag{25}$$

Nonzero solutions $\mathscr{E}'(\mathbf{r}_a) \neq 0$ of Eq. (24) for the incoming wave equal to zero $\mathscr{E}^{(0)}(\mathbf{r}_a) \equiv 0$ may be interpreted as localized waves.[4] Let us stress that perfectly localized waves exist only in *infinite* systems of dipoles.[4] For

example, this might be an infinite periodic lattice of dipoles with one node removed, i.e., a photonic crystal with wave localized around *isolated* point defect.

4. ANDERSON LOCALIZATION

It seems reasonable to expect that each electromagnetic wave localized in a system of dipoles (9) usually corresponds to a certain *curve* on the plane $\{\omega, \phi\}$. Nevertheless, in the case of *random* and infinite system of dipoles there can exist an entire continuous band of spatially localized states corresponding to a region in the plane $\{\omega, \phi\}$. After choosing a point (ω, ϕ) from this region a localized wave of frequency (arbitrarily near) ω exists in almost any random distribution of the dipoles described by the scattering properties ϕ. To illustrate this statement we have to study the properties of *finite* systems for increasing number of dipoles N (while keeping the density constant). For each distribution of the dipoles $\mathbf{r}_a$ placed randomly inside a sphere with the uniform density $n = 1$ dipole per wavelength cubed we have diagonalized numerically the $\tilde{M}$ matrix from Eq. (24) and obtained the lowest eigenvalue:

$$\Lambda(\phi) = \min_j |\lambda_j(\phi)| \tag{26}$$

The resulting probability distribution $P_\phi(\Lambda)$, calculated from 10^3 different distributions of N dipoles is normalized in the standard way

$$\int d\Lambda \, P_\phi(\Lambda) = 1 \tag{27}$$

Let us now compare the surface plots of $P_\phi(\Lambda)$ (treated as a function of two variables ϕ and Λ) calculated for systems consisting of $N = 100$ and 1000 dipoles. They are presented in Figs. 1 and 2, respectively. It is seen from inspection of these plots that, for increasing size of the system (in our case it increased $\sqrt[3]{10} \simeq 2$ times), at some ϕ the probability distribution $P_\phi(\Lambda)$ apparently moves towards the $\Lambda = 0$ axis and simultaneously its variance decreases. This happens only for values of $|\phi|$ that are sufficiently greater than zero. Our numerical investigations indicate that in the limit of an infinite medium, the probability distribution presented in these figures tends to the delta function

$$\lim_{N \to \infty} P_\phi(\Lambda) = \delta(\Lambda), \qquad \text{for} \quad |\phi| \geqslant \phi_{cr} \tag{28}$$

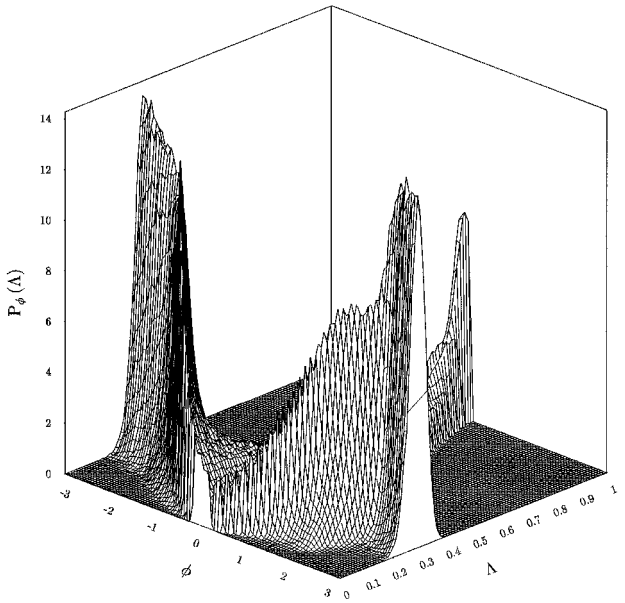

Fig. 1. Surface plot of the probability distribution $P_\phi(\Lambda)$ calculated for 10^3 systems of $N = 100$ dielectric spheres distributed randomly in a sphere with the uniform density $n = 1$ sphere per wavelength cubed.

Therefore, for some ϕ the function $\Lambda(\phi)$ turns out to be a self-averaging quantity. This means that for almost any random distribution of the dipoles $\mathbf{r}_a$, the equation $\lambda_j(\phi) = 0$ holds. Thus, as we expected, a localized wave (described by the corresponding eigenvector of the $\bar{M}$ matrix) exists.

Similarly, localized electronic states in solids appear always at discrete energies only. However, in the case of a *disordered* and *unbounded* system a countable set of energies corresponding to localized states becomes dense in some finite interval (in the same way as the rational numbers are dense in the set of real numbers[39]). When this happens, Anderson localization occurs. But it is always difficult to distinguish between the allowed energies which may be arbitrarily close to each other (by convention the spectrum is a coarse-grained object[39]). Therefore, physically speaking, an entire *continuous* band of spatially localized electronic states exists.

We see from Eqs. (23) and (28) that the total scattering cross section of individual particles σ must exceed some critical value $\sigma_{\mathrm{cr}} = \sigma(\phi_{\mathrm{cr}})$ before localization will take place in the limit $N \to \infty$. This fact is in perfect agreement with the scaling theory of localization[40]: In 3D random media a certain critical degree of disorder is needed for localization. Moreover, our

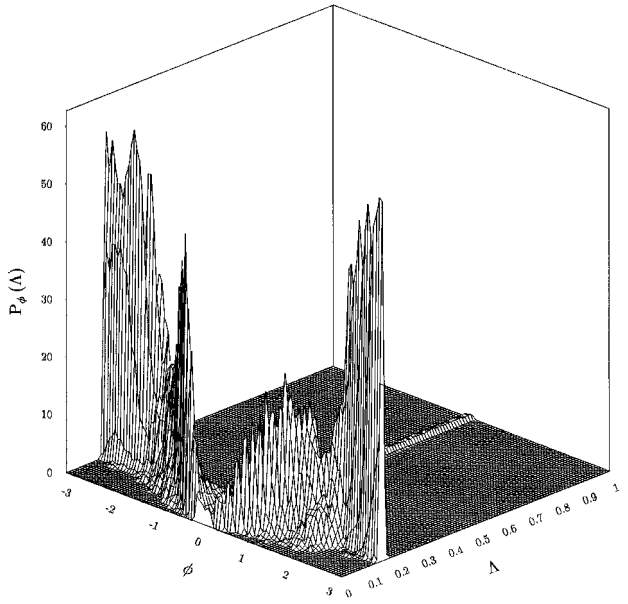

Fig. 2. Same as in Fig. 1 but for 10^3 systems of $N = 1000$ spheres.

preliminary calculations indicate that the value of $k^2\sigma_{cr}$ may decrease with n but *slower* than n^{-2}. Using the Rayleigh expression for the total scattering cross-section σ of a dielectric sphere with radius a and dielectric constant ε[33]:

$$k^2\sigma = \frac{8}{3\pi}(ka)^6 \left|\frac{\varepsilon - 1}{\varepsilon + 1}\right|^2 \tag{29}$$

we conclude that in the long-wave limit the system of dielectric spheres distributed with constant absolute density $\eta = k^3 n/(2\pi)^3$ will be out of the localization regime. On the other hand, in the limit of small wavelengths, the propagation of light is ruled by the laws of geometrical optics, and the point-scatterer approximation we use becomes invalid. Therefore our results seem to agree with the common believe (see, e.g., Refs. 22 and 23), that in three-dimensional media Anderson localization of light is possible only in a certain frequency window.

If the total scattering cross-section of individual particles exceeds some critical value $\sigma \geqslant \sigma_{cr} > 0$, then the scattering mean free path

$$kl = \frac{(2\pi)^3}{(k^2\sigma)\, n} \geqslant \frac{8\pi^2}{3n} \tag{30}$$

(to prove the inequality we used Eq. (23)) must be smaller than the corresponding critical value $l_{cr} = l(\sigma_{cr})$ before localization will set in. In solid state physics a condition $kl \leqslant 1$ applies.[7, 9, 41] However, it follows from Eq. (30) that the standard Ioffe–Regel criterion of localization $kl_{cr} = 1$ is impossible to satisfy in our model for a scaled density $n = 1$ (typically encountered in experiments). On the other hand we have just shown that the Anderson localization could be possible at this density. This means that in the case of electromagnetic waves the localization transition can occur much earlier than predicted by the standard Ioffe–Regel criterion. Why then is there still no convincing experimental demonstration that strong localization could be possible in 3D random dielectric structures (where we have $kl = 10$)?

Let us stress that from the practical point of view the Ioffe–Regel criterion for localization is a very delicate one to achieve. The mean free path $l = 1/(\eta\sigma)$ decreases both with increasing absolute density η of the medium and the scattering cross-sections of the spheres σ. However, the density should be low enough to consider the dielectric particles as randomly placed. Therefore to increase the scattering cross-sections the experimentalists usually tune the frequency of light to match the internal (Mie-type) resonances of individual scatterers. There was considerable controversy some time ago about the role of resonant scattering in combination with localization.[42-44] Within our approach internal resonances of scatterers can be modeled by $|\phi| \simeq \pi$. Our calculations do not exclude the possibility that in *infinite* medium the band of localized waves may appear in this region of ϕ. However, in all experiments we can investigate only systems confined to certain *finite* regions of space. And, as follows from Figs. 1 and 2, the band of localized waves appears *faster* with increasing size of the system when $\phi_{cr} \leqslant |\phi| \ll \pi$, i.e., when the frequency is *not* tuned to the internal resonances of individual scatterers.

Let us emphasize that this result is specific for 3D random media. In both one and two dimensions, macro- and microscopic resonances appear at the same frequencies. To illustrate this fact, in Fig. 3 we have prepared a plot analogous to Fig. 1 but calculated for 10^3 configurations of $N = 100$ dielectric cylinders modeled by 2D dipoles.[1] In this case the band of localized waves does appear faster for $|\phi| \simeq \pi$: in 2D the parameters of the single scatterers that give the internal and global resonances coincide, and matching the internal resonances helps to establish localization. In our opinion this could be the main reason why a convincing experimental demonstration of strong localization of microwave radiation has been given only for two dimensions,[25] although also more practical reasons as, e.g., polydispersity of the actual 3D samples (leading to phase shifts) can play a role. Results obtained from our model for 2D media (which seem to

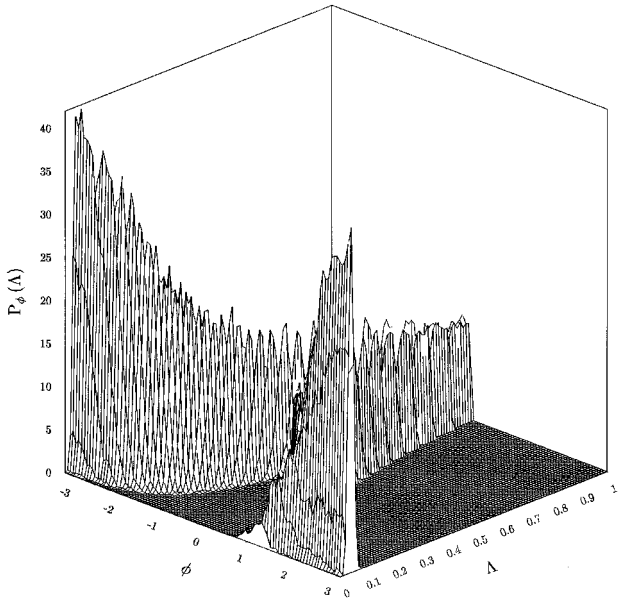

Fig. 3. Surface plot of the probability distribution $P_\phi(\Lambda)$ calculated for 10^3 systems of $N = 100$ dielectric cylinders distributed randomly in a circle with the density $n = 1$ cylinder per wavelength squared.

agree with experimental results) prove that the surprising features of localization we observed for 3D random media are *not* the artifacts produced by the model.

5. BRIEF SUMMARY

We have developed an effective theoretical approach (based on analysis of probability distributions) to Anderson localization of electromagnetic waves in random distributions of dielectric spheres. Investigating spectra of certain random matrices we have actually observed numerically some indications of appearance of the continuous *band* of localized electromagnetic waves emerging in the limit of the infinite medium. This approach can provide deeper insight into the existing experimental results. We showed that it should be possible to observe strong localization in 3D random dielectric media without satisfying the heuristic Ioffe–Regel criterion. But to achieve this Holy Grail the experiments should be performed for a different range of parameters than is presently considered to be promising

for localization. In three-dimensional random media the cross sections of single scatterers should *not* be made maximal. The reason is that (as opposed to 2D) the global resonances of the 3D samples of scatterers are formed for different parameters than resonances of the single scatterer.

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