ANDERSON LOCALIZATION OF ELECTROMAGNETIC WAVES IN CONFINED DISORDERED DIELECTRIC MEDIA

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1. Introduction

Scattering of electromagnetic waves from varoius kind of obstacles is rich of intresting and sometimes unexpected phenomena. Already for two scatters placed together well within a wavelentg an extremely narrow proximity resonance can appear [10]. For many randomly distributed scatters we may expect that, for same range of parameters, Anderson localization can show up.

A convincing experimental demonstration that Anderson localization of electromagnetic waves is possible in three-dimensional disordered dielectric structures has been given recently [12]. The strongly scattering medium has been provided by semiconductor powders with a very large refractive index. By decreasing the average particle size it was possible to observe a clear transition from linear scaling of transmission ($T \propto L^{-1}$) to an exponential decay ($T \propto e^{-L/\xi}$). Some localization effects have been also reported in previous experiments on microwave localization in copper tubes filled with metallic and dielectric spheres [4]. However, the latter experiments were plagued by large absorption, which makes the interpretation of the data quite complicated.

Another experiment on microwave localization has been performed in a two-dimensional medium [1]. The scattering chamber was set up as a collection of dielectric cylinders randomly placed between two parallel aluminum plates on half the sites of a square lattice. These authors attributed the observed sharp peaks of transmission to the existence of localized modes and measured the energy density of the electromagnetic field localized by their random structures.

As shown in our previous paper [7], Anderson localization of classical waves can be studied theoretically by investigating a striking phase transition in the spectra of certain random matrices. The elements whose elements are equal to the Green's function calculated for the differences between positions of any pair of scatterers. The Breit-Wigner's model of a single scatterer allows one to give a sound physical interpretation to the universal properties of these matrices. In this case the spectrum can be considered as an approximation to the resonance poles of the system. In the limit of an infinite random medium all eigenvalues condense to a smooth line instead of redistributing themselves over the complex plane. Physically speaking this corresponds to the formation of an entire frequency band of spatially localized electromagnetic waves.

The Green's function is one of the fundamental basic building blocks for constructing a self-consistent description of multiple scattering processes. In the free-space case the Green's function is something very simple. It describes a spherical (in 3D), cylindrical (in 2D) or a pair of two plane (in 1D) outgoing wave(s) centered at the scatterers position. Boundary conditions may change the Green's function in a complicated way. It is therefore interesting to extend the random-matrix description of localization to encompass the case of nontrivial boundary conditions. For this purpose in this paper we investigate Anderson localization of electromagnetic waves in a disordered dielectric medium confined within a metallic waveguide. The results of our previous paper [9] are extended to the case of a multimode waveguide and new physical interpretation based on the Breit-Wigner's model of a single scatterer is presented.

2. Basic assumptions

In the following we study the properties of the stationary solutions of the Maxwell equations in two-dimensional media consisting of randomly placed parallel dielectric cylinders of infinite height (i.e., very long as compared to the wavelength of the electromagnetic field). This means that one, say (y), out of three dimensions is translationally invariant and only the remaining two (x, z) are random. In the present model we place the disordered dielectric medium between two infinite, perfectly conducting mirrors described by the equations $x = 0$ and $x = d$. Thus we will consider the case where the cylinders are oriented parallel to the mirrors. For simplicity our discussion will be restricted to TE modes polarized along the y axis only. The main advantage of this two-dimensional approximation is that we can use the scalar theory of electromagnetic waves [8]:

$$\vec{E}(\vec{r}, t) = \mathrm{Re}\left\{\vec{e}_y\, \mathcal{E}(x, z)\, e^{-i\omega t}\right\}. \tag{1}$$

Consequently, the polarization of the medium takes the form:

$$\vec{P}(\vec{r}, t) = \mathrm{Re}\left\{\vec{e}_y\, \mathcal{P}(x, z)\, e^{-i\omega t}\right\}. \tag{2}$$

Localization of electromagnetic waves in disordered 2D media is usually studied experimentally in microstructures consisting of dielectric cylinders with diameters and mutual distances being comparable to the wavelength [1]. However it seems to be a reasonable assumption that what really counts for the basic features of localization is the scattering cross-section and not the real geometrical size of the scatterer itself. Therefore we will represent the dielectric cylinders located at the points (x_a, z_a) by single 2D electric dipoles:

$$\mathcal{P}(x, z) = \sum_{a=1}^{N} p_a \, \delta^{(2)}(x - x_a, z - z_a). \tag{3}$$

It should be stressed, that the boundary conditions considered in this paper are different from those encountered in the experiment of Ref. [1]. To minimize the effect of the waves reflected off the edges of the scattering chamber, its perimeter was lined with a layer of microwave absorber. Therefore, to model that particular experiment, it is appropriate to use the free space boundary conditions (as we did in our previous papers [8, 11]).

3. Planar waveguide

Several particular cases may be considered ($k\,d = \pi$ is the cut-off thickness of the waveguide). For $N = 0$ and $k\,d < \pi$ there are no guided modes in the waveguide as well as there are no localized waves. This case is analogous to the electronic band gap in a solid. If $N > 0$ and $k\,d < \pi$ there are still no guided modes in the waveguide but localized waves can appear for any distribution of the cylinders. It is again analogous to the solid state physics situation where isolated perturbations of the periodicity of crystals (like impurities or lattice defects) can lead to the formation of localized electronic states with energies within the forbidden band. Another possibility corresponds to $N = 0$ and $k\,d > \pi$. In this case there are guided modes but the system supports no localized waves. This is very similar to the conductance band in solids. Guided modes correspond to extended electronic states described by Bloch functions. In this paper we perform a detailed study of the regime where $N > 0$ and $k\,d > \pi$. For this range of parameters there are both the guided modes and the resonances of transmission. Isolated localized waves can be seen for certain distributions of the cylinders. The signs of Anderson localization emerging in the limit of an infinite medium can be observed both in analysis of transmission and in the properties of the spectra of certain random matrices. Eventually we will consider a limiting case of $N \to \infty$ and $k\,d > \pi$. It turns out that in this case the guided modes no longer exist in the waveguide. Instead a band of localized waves will be formed for any distribution of the cylinders. It is an interesting analog of the Anderson localization in noncrystalline solids such as amorphous semiconductors or disordered metals.

4. Field expansion

The electric field of the electromagnetic wave incident on the cylinders

$$\mathcal{E}^{(0)}(x,z) = \sum_{m=1}^{M} \iota_m \, \mathcal{E}^{(m)}(x,z), \tag{4}$$

may be expanded into the guided modes of the waveguide [5]:

$$\mathcal{E}^{(m)}(x,z) = \frac{2}{\sqrt{\beta_m \, d}} \, \sin(\alpha_m \, x) \, e^{i\beta_m \, z}, \tag{5}$$

where the propagation constants are given by:

$$\alpha_m = \frac{\pi}{d} \, n, \quad \beta_m = \sqrt{k^2 - \alpha_m^2}. \tag{6}$$

The total field that can be measured *far* from the cylinders is fully described by the reflection ρ_m and transmission τ_m coefficients of all guided modes:

$$\mathcal{E}(x,z) = \sum_{m=1}^{M} \iota_m \, \mathcal{E}^{(m)}(x,z) + \sum_{m=1}^{M} \rho_m \, \mathcal{E}^{(m)*}(x,z) \quad \text{for} \quad z \to -\infty \tag{7}$$

$$\mathcal{E}(x,z) = \sum_{m=1}^{M} \tau_m \, \mathcal{E}^{(m)}(x,z) \quad \text{for} \quad z \to +\infty \tag{8}$$

Using the Lorentz theorem and repeating the straightforward but lengthly calculations (see, e.g., [5]) we easily arrive at the following expressions determining the transmission coefficients

$$\tau_m = \iota_m + i\pi \, k^2 \sum_{a=1}^{N} p_a \, \mathcal{E}^{(m)*}(x_a, z_a), \tag{9}$$

and the reflection coefficients

$$\rho_m = i\pi \, k^2 \sum_{a=1}^{N} p_a \, \mathcal{E}^{(m)}(x_a, z_a), \tag{10}$$

for a given dipole moments p_a. In the following sections we will relate p_a to the values of the incident field calculated at the positions of the cylinders $\mathcal{E}^{(0)}(x_a, z_a)$.

5. Method of images

A simple way to take into account the boundary conditions of parallel mirrors and their influence on the electromagnetic field is to use the method of images. This technique has been used, i.e., in QED calculations of spontaneous emission in cavities [3, 2]. To reproduce the correct boundary conditions on the radiation field of each cylinder (3) the mirrors are replaced by an array of image cylinders whose phases alternate in sign:

$$\mathcal{P}(x,z) = \sum_{a=1}^{N} \sum_{j=-\infty}^{\infty} (-1)^j p_a \, \delta^{(2)}(x - x_a^{(j)}, z - z_a), \tag{11}$$

where

$$x_a^{(j)} = (-1)^j x_a + jd. \tag{12}$$

Thus a finite system of dielectric cylinders (3) placed within a metallic waveguide is fully equivalent to an infinite system of cylinders (11) forming a slab in a free space. This fact allows us to utilize some results from our previous paper concerning dielectric cylinders in free space [8].

6. Elastic scattering

It is now a well-established fact that to use safely the point-scatterer approximation it is essential to use a representation for the cylinders that conserves energy in the scattering processes. In the case of a system of cylinders in free space this requirement means that for each cylinder the optical theorem holds [8]:

$$\pi \, k^2 \, |p_a|^2 = \mathrm{Im} \left\{ p_a^* \, \mathcal{E}'(x_a, z_a) \right\}. \tag{13}$$

Eq. (13) gives the following form of the coupling between the dipole moment p_a and the electric field incident on the cylinder $\mathcal{E}'(x_a, z_a)$:

$$i\pi \, k^2 \, p_a = \frac{e^{i\phi} - 1}{2} \, \mathcal{E}'(x_a, z_a), \tag{14}$$

The same result holds also for a system of cylinders placed in a metallic waveguide (11).

7. Multiple scattering

In the case of a confined medium the field acting on the ath cylinder $\mathcal{E}'(x_a, z_a)$ from Eq. (14) is the sum of the incident guided mode $\mathcal{E}^{(0)}$, which obeys the Maxwell equations in an empty waveguide, and waves scattered by all other cylinders *and* by all images:

$$\mathcal{E}'(x_a, z_a) = \mathcal{E}^{(0)}(x_a, z_a) + \frac{e^{i\phi} - 1}{2} \sum_{b=1}^{N} G_{ab}\, \mathcal{E}'(x_b, z_b), \quad a = 1, \dots N \tag{15}$$

Thus, in the present model the G matrix from Eq. (15) needs to be defined differently than in Ref. [8]:

$$i\pi\, G_{ab} = 2 \sum_{\rho_{ab}^{(j)} \neq 0} (-1)^j\, K_0(-ik\rho_{ab}^{(j)}), \tag{16}$$

where

$$\rho_{ab}^{(j)} = \sqrt{(x_a - x_a^{(j)})^2 + (z_a - z_b)^2}, \tag{17}$$

denotes the distance between the ath cylinder and the jth image of the bth cylinder and K_0 is the modified Bessel function of the second kind. Note that summation in Eq. (16) is performed over all j, for which $\rho_{ab}^{(j)} \neq 0$.

The system of linear equations (15) fully determines the field acting on each cylinder $\mathcal{E}'(x_a, z_a)$ for a given field of the guided mode $\mathcal{E}^{(0)}(x_a, z_a)$ incident on the system. Analogous relationships between the stationary outgoing wave and the stationary incoming wave are known in the general scattering theory as the Lippmann-Schwinger equations [6]. If we solve Eqs. (15) and use Eqs. (14) to find p_a, then we are able to find the transmission and reflection coefficients given by Eqs. (9) and (10).

8. Reflection coefficient

Substituting Eq. (14) into Eq. (10) we get the following expression for the reflection coefficient of a mth guided mode from a system of N identical cylinders:

$$|\rho_m|^2 = \sin^2 \frac{\phi}{2} \left| \sum_{a=1}^{N} \mathcal{E}'(x_a) \cdot \mathcal{E}^{(m)}(x_a, y_a) \right|^2 \tag{18}$$

If $\mathcal{E}'(x_a)$ is an eigenvector of the G matrix (16) corresponding to the eigenvalue λ

$$\sum_{b=1}^{N} G_{ab}\, \mathcal{E}'(x_b, y_b) = \lambda\, \mathcal{E}'(x_a, y_a), \quad a = 1, \dots, N \tag{19}$$

then the reflection coefficient takes the form of

$$\begin{aligned} |\rho_m|^2 \;\; &= \;\; \frac{1}{(\cot\phi + \operatorname{Im}\lambda)^2 + (1 + \operatorname{Re}\lambda)^2} \\ &\times \;\; \left| \sum_{a=1}^{N} \mathcal{E}^{(0)}(x_a, y_a) \cdot \mathcal{E}^{(m)}(x_a, y_a) \right|^2 \end{aligned} \tag{20}$$

Only the first term in Eq. (20) depends on the model of the scatterer (through the phase shift ϕ). The incident field appears only in the second term. Both terms depend on the geometry of the system (the first one through the eigenvalue λ) and frequency.

9. Breit-Wigner scatterers

The cylinders necessarily have an internal structure. Thus in general the phase shift ϕ should be regarded as a function of frequency ω. For example to model a simple scattering process with one internal Breit-Wigner type resonance one can write:

$$\cot \frac{\phi(\omega)}{2} = -\frac{\omega - \omega_0}{\gamma_0} \tag{21}$$

The total scattering cross-section $k\,\sigma = 4\,\sin^2 \phi/2$ [8] takes then the familiar Lorentzian form:

$$k\,\sigma = \frac{4\,\gamma_0^2}{(\omega - \omega_0)^2 + \gamma_0^2} \tag{22}$$

Substituting the Breit-Wigner model of scattering from Eq. (21) into Eq. (20) we get the following expressions for the reflection coefficients of the guided modes:

$$\begin{aligned} |\rho_m(\omega)|^2 &= \overbrace{\frac{\gamma_0^2}{[\omega - (\omega_0 + \gamma_0 \,\mathrm{Im}\,\lambda)]^2 + [\gamma_0(1 + \mathrm{Re}\,\lambda)]^2}}^{\text{rapidly varying}} \\ &\times \underbrace{\left| \sum_{a=1}^{N} \mathcal{E}^{(0)}(x_a, y_a) \cdot \mathcal{E}^{(m)}(x_a, y_a) \right|^2}_{\text{slowly varying}} \end{aligned} \tag{23}$$

Most localization experiments are performed in the range of optical or microwave frequencies. In this case usually $\gamma_0/\omega_0 \ll 1$. It is therefore reasonable to assume that the first term in Eq. (23) varies with frequency much faster than the second one. Thus we get a Lorentzian-type resonance of width

$$\gamma \simeq \gamma_0(1 + \mathrm{Re}\,\lambda) \tag{24}$$

centered around frequency

$$\omega \simeq \omega_0 + \gamma_0 \,\mathrm{Im}\,\lambda \tag{25}$$

10. Single scattering

Let us begin by considering a single dielectric cylinder placed in a metallic waveguide. In the following we will assume that only *one* guided mode exists. The scattering is elastic. Thus on average the energy scattered by the cylinder must be equal to the energy given to the cylinder by the incident wave. Therefore:

$$|\tau_1|^2 + |\rho_1|^2 = 1 \tag{26}$$

Substituting into Eq. (26) the formulas for the transmission and reflection coefficients Eqs. (9) and (10) we get the following form of the coupling between the dipole moment p_1 of the cylinder and the electric field of the incident wave calculated at the cylinder $\mathcal{E}^{(0)}(x_1, z_1)$:

$$i\pi\, k^2\, p_1 = \frac{e^{i\phi'} - 1}{2}\, \mathcal{E}^{(0)}(x_1, z_1), \tag{27}$$

Thus all the scattering properties of a cylinder are perfectly described by a phase shift ϕ'. The explicit form of the transmission and reflection coefficients from a single cylinder reads than as:

$$\tau = \frac{e^{i\phi'} + 1}{2}, \quad \rho = \frac{e^{i\phi'} - 1}{2} \tag{28}$$

Notice please that the waveguide may introduce it's own phase shift depending on the position of the cylinder with respect to its walls. Therefore the free space phase shift ϕ from Eq. (21) is different from the phase shift ϕ' from Eq. (27). This results in changing the width and position of the transmission and reflection resonance.

Indeed, as an example let us consider a cylinder placed between the mirrors separated by a distance $k\,d = 3\pi/2$. We have calculated numerically the corresponding 1×1 G-matrix (16). The images of the cylinder from Eq. (11) were summed from $j = -50000$ to $j = 50000$. The resulting "spectrum" ($\lambda = G_{11}$) of the G-matrix is ploted on the left side of Fig. 1. We see, that as opposed to the free-space case ($G_{11} = 0$) it is nonzero.

According to Eqs. (24) and (25) the eigenvalues λ can be considered as a first-order approximation (in γ_0/ω_0) to the positions of the resonance poles in the system. Thus if $\lambda \neq 0$ then $\omega \neq \omega_0$, $\gamma \neq \gamma_0$. To check if the shape of the resonance has really changed on the right side of Fig. 1 we have ploted as a solid line the reflection coefficient $R = |\rho_1|^2$. The dashed line on the same plot corresponds to the "free space" case $\lambda = 0$.

11. Collective resonances

In the next step in Figs. 2 and 3 we plot the reflection R of the systems of $N = 10, 100$ cylinders as a function of the frequency ω. In the same plots

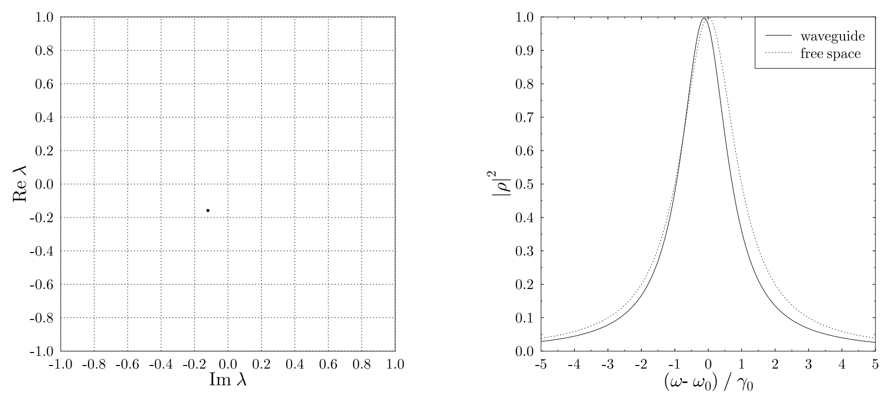

Figure 1. Reflection R of a single dielectric cylinders placed in a planar metallic waveguide ploted as a function of the frequency ω and the corresponding spectrum of the G matrix. The dashed line correspond to the transmission in a "free space" case.

we have also the corresponding approximate values of the resonance poles given by the spectrum of the G-matrix.. The cylinders were distributed randomly with constant uniform density $n = 1$ cylinder per wavelength squared. Therefore for each N the size of the system was proportional to the number of cylinders $L \propto N$.

We have seen that in the case $N = 1$ the incident wave was totally reflected for a *single* value of $\omega = \tilde{\omega}$. Note that not necessarily $\tilde{\omega} = \omega_0$, and therefore for this value of ω the total scattering cross-section σ of an individual dielectric cylinder (22) does not approach its maximal value. However, for systems containing $N = 10$ and $N = 100$ the entire *regions* of the values of frequencies ω exist for which $R \simeq 1$. They are separated by narrow maxima of transmission. Moreover, inspection of Figs. 2 and 3 suggests that in the limit $N \to \infty$ the number of these maxima increases and simultaneously they became narrower and sharper. Therefore we may expect that for sufficiently large N the incident waves will be totally reflected for almost any ω except the discrete set $\omega = \omega_l$ for which the reflection is close to unity. Physically speaking this means that different realizations of sufficiently large system of randomly placed cylinders are hardly distinguishable from each other by a transmission experiment.

It follows from inspection of Eqs. (9) and (10) that the maximum of transmission $T = \sum_m |\tau_m|^2 = 1$ (and minimum of reflection $R = \sum_m |\rho_m|^2 = 0$, because the medium is non-dissipative) corresponds to the

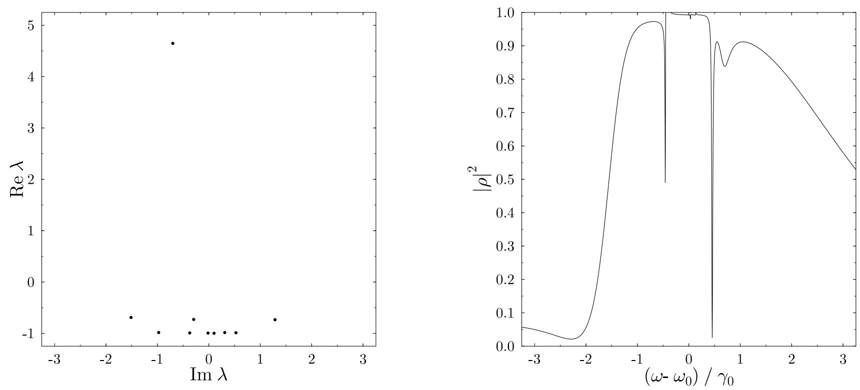

Figure 2. On the right: reflection R of q system of $N = 10$ dielectric cylinders placed randomly in a planar metallic waveguide ploted as a function of the frequency ω. On the left: first-order approximations to the resonance poles in the system given by the spectrum λ of the corresponding G-matrix.

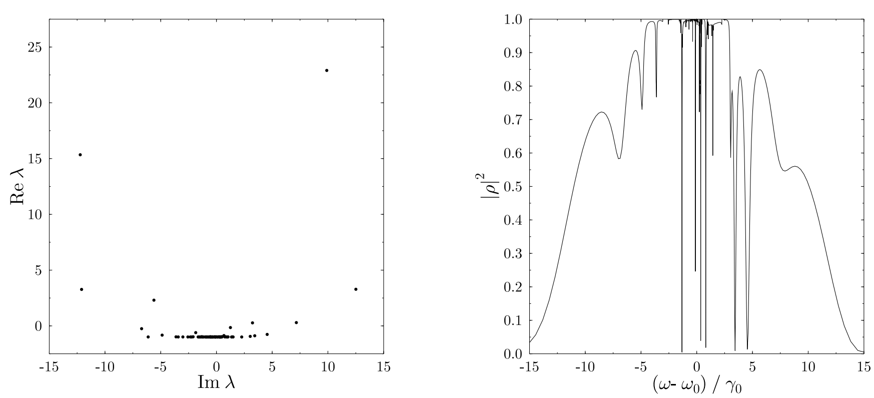

Figure 3. On the right: reflection R of q system of $N = 10$ dielectric cylinders placed randomly in a planar metallic waveguide ploted as a function of the frequency ω. On the left: first-order approximations to the resonance poles in the system given by the spectrum λ of the corresponding G-matrix.

case when the polarization of the medium fulfills the following condition:

$$\sum_{a=1}^{N} p_a \, \mathcal{E}^{(0)*}(x_a, z_a) = \sum_{a=1}^{N} p_a \, \mathcal{E}^{(0)}(x_a, z_a) = 0 \quad \text{for} \quad m = 1, \dots M \qquad (29)$$

This means that in the expansion of the field radiated by the medium into waveguide modes the coefficients near *all* guided modes Eq. (5) vanish. Therefore the radiated field consists only of evanescent modes with imaginary propagation constants β_m and thus it is exponentially localized in the vicinity of the medium. In the next section we will show that such a field can exist also without any incident wave and therefore represents a truly *localized* wave.

12. Localized waves

By definition, an electromagnetic wave is localized in a certain region of space if its magnitude is (at least) exponentially decaying in any direction from this region. We will show now that electromagnetic waves localized in the system of dielectric cylinders placed in a planar metallic waveguide correspond to nonzero solutions $\mathcal{E}'_l(x_a, z_a) \neq 0$ of Eqs. (15) for the incoming wave equal to zero, i.e., $\mathcal{E}^{(0)}(x, z) \equiv 0$. Note that we added an index l which labels the localized waves.

Indeed, let us suppose that the field is exponentially localized. This means that there are no guided modes in the radiation field. Therefore (as shown in the previous section) Eq. (29) holds. Using Eq. (14) we see that the vector formed by the values of the field acting on the cylinders is *orthogonal* to the vector formed by the values of incident field calculated at the positions of the cylinders:

$$\sum_{a=1}^{N} \mathcal{E}'_l(x_a, z_a)\, \mathcal{E}^{(0)*}(x_a, z_a) = 0. \tag{30}$$

But simultaneously $\mathcal{E}'_l(x_a, z_a)$ is a solution of a system of linear Eqs. (15) where $\mathcal{E}^{(0)}(x_a, z_a)$ is the right-hand-side. Therefore $\mathcal{E}'_l(x_a, z_a)$ is also a solution of Eqs. (15) with $\mathcal{E}^{(0)}(x_a, z_a) \equiv 0$.

The proof works also the other way round. Suppose that $\mathcal{E}'_l(x_a, z_a)$ is a solution of Eqs. (15) for $\mathcal{E}^{(0)}(x_a, z_a) \equiv 0$. As the considered medium is non-dissipative, the time average energy stream integrated over a closed surface surrounding it must vanish. This means that there are again no guided modes in the radiation field (which in the case $\mathcal{E}^{(0)}(x, z) \equiv 0$ is equal to the total field). Therefore Eq. (29) holds and the wave is localized.

13. Eigenproblem

Note that for the incoming wave equal to zero, i.e., $\mathcal{E}^{(0)}(x,z) \equiv 0$, the system of equations (15) is equivalent to the *eigenproblem* for the G matrix:

$$\sum_{b=1}^{N} G_{ab}\,\mathcal{E}'_l(x_b,z_b) = \lambda_l\,\mathcal{E}'_l(x_a,z_a), \tag{31}$$

where

$$\lambda_l = -1 - i\,\cot\frac{\phi}{2}. \tag{32}$$

Let us stress that Eq. (32) can be fulfilled *only* if the real part of an eigenvalue satisfies:

$$\mathrm{Re}\,\lambda_l = -1. \tag{33}$$

Substituting the Breit-Wigner model of scattering (21) we see that the imaginary part of the eigenvalue and the frequency of the localized wave are then related by:

$$\mathrm{Im}\,\lambda_l = \frac{\omega_l - \omega}{\gamma_0}. \tag{34}$$

Therefore only those eigenvectors $\mathcal{E}'_l(x_a,z_a)$ of the G matrix which correspond to the eigenvalues λ_l satisfying the condition (33) may be related to localized waves. Moreover, those waves can exist only for discrete frequencies ω_l given by Eq. (34). As discussed in the previous sections they are the same values of ω for which the transmission is equal to unity.

14. Anderson localization

Note that in finite dielectric media no localized states are supported by Maxwell's equations in two dimensions [8]. However, this is not the case with confined media, where localized waves do exist even in finite media. Therefore a clear distinction between localized waves with isolated frequencies and the dense band of localized waves (due to Anderson localization) is needed. We show now that this distinction may be provided by investigation of a phase transition which occurs in the limit of $N \to \infty$ in the spectra of G_{ab} matrices corresponding to systems of randomly distributed dielectric cylinders.

To support this statement let us recall Figs. 2 3. We have ploted there the spectra λ of a G matrix (diagonalized numerically) corresponding to certain specific configurations of $N = 10$ and $N = 100$ cylinders placed randomly with the uniform density $n = 1$ cylinder per wavelength squared. We see that already in the case of of $N = 10$ quite a lot of eigenvalues

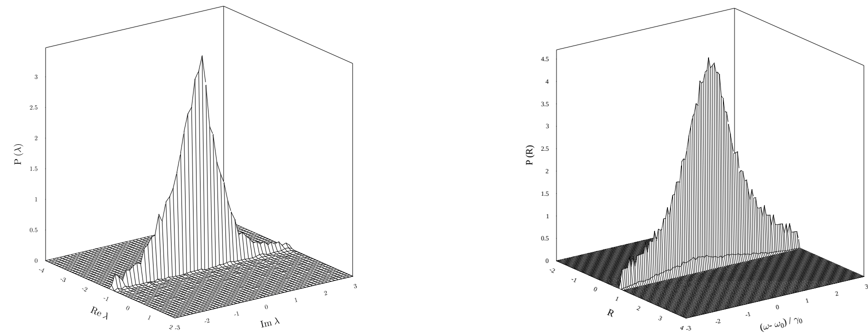

Figure 4. On the right: density of eigenvalues $P(\lambda)$ of the matrix G calculated from 10^2 distributions of $N = 100$ cylinders. On the left: probability $P(R)$ of measuring a reflection R at frequency ω calculated for the same systems. The similarity is striking.

are located near the $\mathrm{Re}\,\lambda = -1$ axis. This tendency is more and more pronounced with increasing size of the system measured by N. This is a universal property of 2D G matrices, not restricted to this specific realization of the system only. To prove these statements we diagonalize numerically the G matrix (16) for 10^2 different distributions of $N = 100$ cylinders. Then we construct two-dimensional histogram of eigenvalues λ from all distributions. It approximates the corresponding probability distribution $P(\lambda)$ which is normalized in the standard way $\int d^2\lambda\, P(\lambda) = 1$. In Fig. 4 we have the surface plot of the function $P(\lambda)$. It clearly shows that for all configurations (without, may be, a set of zero measure) most eigenvalues are located near the $\mathrm{Re}\,\lambda = -1$ axis.

Our numerical investigations indicate that in the limit of an infinite medium, the probability distribution under consideration will tend to the delta function in the real part:

$$\lim_{N\to\infty} P(\lambda) = \delta(\mathrm{Re}\,\lambda + 1)\, f(\mathrm{Im}\,\lambda). \tag{35}$$

This means that in this limit for almost any random distribution of the cylinders, an infinite number of eigenvalues satisfies the condition Eq. (33). It is therefore reasonable to expect that in the case of a random and infinite system a countable set of frequencies ω_l corresponding to localized waves becomes *dense* in some finite interval. But it is always difficult to separate such frequencies from frequencies which may be arbitrarily near and physically the spectrum is always a coarse-grained object. Therefore in the limit of an infinite medium an entire *band* of spatially localized electromagnetic waves appears.

In addition in Fig. 4 we have the probability $P(R)$ of measuring a reflection R at frequency ω calculated for the same systems of cylinders. This distribution is very similar to the distribution of eigenvalues. Thus incident waves are totally reflected for "almost any" frequency from the band of localized waves, i.e., except the discrete set (of zero measure) for which the transmission is equal to unity. This provides a connection between the phenomenon of localization and a dramatic inhibition of the propagation of electromagnetic waves in a spatially random dielectric medium.

15. Summary

In summary, we have developed a simple yet reasonably realistic theoretical approach to Anderson localization of electromagnetic waves in two-dimensional dielectric media confined within a metallic waveguide. The results of our previous papers dealing with the free space configurations are now extended to encompass the case of nontrivial boundary conditions. A sound physical interpretation in terms of transmission experiment is also proposed. By confining a system of randomly distributed dielectric cylinders into a planar metallic waveguide we are able to observe clear signs of Anderson localization already for $N = 100$ scatterers. One of the indicators of localization is the phase transition in the spectra of certain random matrices. This property of random Green functions is now generalized to the case of a confined dielectric medium (where the Green function is different). It may be interpreted as an appearance of the band of localized electromagnetic waves emerging in the limit of the infinite medium. A connection between this phenomenon and a dramatic inhibition of the propagation of electromagnetic waves in a spatially random dielectric medium was provided. A clear distinction between isolated localized waves (which do exist in finite confined media) and the band of localized waves (which appears in the limit of an infinite random medium) was also presented.

16. Acknowledgments

The authors are grateful to Sergey Skipetrov and Bart van Tiggelen for the kind hospitality in Cargese. This work was supported in part by Polish KBN grant No. 2 P03B 044 19.

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